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ScienceDirect
Computational Materials Science 41 (2008) 322–329
COMPUTATIONAL MATERIALS SCIENCE
www.elsevier.com/locate/commatsci

Two-dimensional boundary element analysis of creep continuum damage problems with plastic effects

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Received 12 January 2007; received in revised form 18 March 2007; accepted 25 April 2007
Available online 6 June 2007

Abstract

In this paper, a quadratic boundary element formulation for elasto-plastic creep damage problems in which strains and deformations are mainly creep dominant is presented. In order to cover both plastic and creep deformations, the initial strain formulation is employed. The Euler method with automatic time-step control scheme is implemented for time integrations. Creep rupture life is predicted using a continuum damage mechanics approach. The proposed formulation is applied to the uniaxial, perforated plate and rotating disk.

Keywords: Boundary element; Creep; Damage; Rupture time; Plasticity

1. Introduction

The boundary element (BE) method is well established as an accurate numerical tool, particularly well suited for linear elastic problems. Due to its high resolution of stresses on the surface, the BE approach has been shown to be accurate in problems involving stress concentration, fracture mechanics and contact analysis. However, its extension to non-linear problems including material and geometric non-linearity is not widespread and is underdeveloped when compared to the finite element (FE). Moreover, there has been very little work on the boundary element implementation of creep continuum damage problems in the literature and BE elasto-plastic creep damage formulations are not covered.

In this paper, a BE formulation for creep damage problems based on an initial strain approach is presented. Constitutive equations using one variable called damage parameter, ω, to characterise the deterioration of the material are employed. Isoparametric quadratic elements are employed for the line elements and surface cells. The objective of this paper is to present the BE method as an alternative accurate approach for elasto-plastic creep damage problems in which strains and deformations are mainly creep dominant.

2. Analytical formulations

2.1. BE analytical formulation

The boundary integral equation for non-linear material behaviour includes an additional domain integral representing the non-linear domain effects. The BE formulation for creep damage is based on the initial strain approach which has the same form as that used for plasticity by replacing plastic strain rates by creep strain rates as follows:

$$\begin{aligned} C_{ij}(P)\dot{u}_i(P) = &-\int_S T_{ij}(P,Q)\dot{u}_i(Q)\mathrm{d}S_Q + \int_S U_{ij}(P,Q)\dot{t}_i(Q)\mathrm{d}S_Q \\ &+\int_A U_{ij}(P,q)\dot{f}_i(q)\mathrm{d}A(q) \\ &+\int_A \left(W_{kij}(P,q) + \overline{W}_{kij}(P,q)\right)\dot{\varepsilon}^c_{ij}(q)\mathrm{d}A(q) \qquad (1) \end{aligned}$$

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doi:10.1016/j.commatsci.2007.04.011

with the following notations: U_{ij}, T_{ij} are the second-order displacement and traction tensors in the i direction at the field point Q or q due to an orthogonal unit load at the variable point P or p in the j direction. $\dot{u}_i, \dot{t}_i$ and $\dot{\varepsilon}_i^c$ are displacement, traction and creep strain rates, respectively. f_i represents body forces. Capital letters are used to indicate that the point concerned lies on the boundary S. Capital letter A represents the solution domain. C_{ij} is the free-term tensor, whose components depend on the geometry, and W_{kij} are the corresponding stress components. These tensors can be derived from the fundamental solution to Kelvin's problem in two dimensions and the auxiliary tensor, $\overline{W}_{kij}$, is given as follows:

$$\begin{aligned}\overline{W}_{kij}(P,q) &= \frac{v}{2\pi(1-v)}\delta_{ij}\frac{1}{r}\frac{\partial r}{\partial x_k} \quad \text{(plane strain)}\\ \overline{W}_{kij}(P,q) &= 0 \quad \text{(plane stress)}\end{aligned} \tag{2}$$

in which r is the distance between P and q. The integral expression for the total strain rates at an interior point p can be obtained by differentiating the corresponding identity for the displacement rate. In the initial strain approach, the convected differentiation of the related domain integrals is employed and at internal points the correct expression for the total strain rates can then be written as follows:

$$\begin{aligned}\dot{\varepsilon}_{ij}(p) = &-\int_S S^{\varepsilon}_{kij}(p,Q)\dot{u}_k(Q)\mathrm{d}S(Q)\\ &+\int_S D^{\varepsilon}_{kij}(p,Q)\dot{t}_k\mathrm{d}S(Q) + \int_A D^{\varepsilon}_{kij}(p,q)\dot{f}_k\mathrm{d}A(q)\\ &+\int_A W^{\varepsilon}_{ijkh}(p,q)\dot{\varepsilon}^c_{kh}\mathrm{d}A(q)\\ &+\int_A \overline{W}^{\varepsilon}_{ijkh}(p,q)\dot{\varepsilon}^c_{kh}(p)\mathrm{d}A(q) + F^{\varepsilon}_{ij}(\dot{\varepsilon}^c_{kh}(p))\end{aligned} \tag{3}$$

in which $D^{\varepsilon}_{kij}, S^{\varepsilon}_{kij}$ and W^{ε}_{ijkh}, are the derivatives of the aforementioned fundamental solutions [1]. The auxiliary tensor (from plastic strain terms in the out-of-plane direction), W^{ε}_{ijkh} is given as

$$\begin{aligned}\overline{W}^{\varepsilon}_{ijkh}(p,q) &= -\frac{v}{4\pi(1-v)}\left(\frac{1}{r^2}\right)\left(\delta_{ij}\delta_{kh} - 2\delta_{ij}\frac{\partial r}{\partial x_k}\frac{\partial r}{\partial x_h}\right)\text{(plane strain)}\\ \overline{W}^{\varepsilon}_{ijkh}(p,q) &= 0 \quad \text{(plane stress)}\end{aligned} \tag{4}$$

The integral free-term, F^{ε}_{ij} depends on the plastic deformation at the load point and it is given by

$$\begin{aligned}F^{\varepsilon}_{ij}(\dot{\varepsilon}^P_{kh}(p)) &= \frac{3-4v}{4(1-v)}\dot{\varepsilon}^P_{kh}(p) - \frac{1}{8(1-v)}\delta_{kh}\dot{\varepsilon}^P_{mm}(p) \quad \text{(plane stress)}\\ F^{\varepsilon}_{ij}(\dot{\varepsilon}^P_{kh}(p)) &= \frac{3-4v}{4(1-v)}\dot{\varepsilon}^P_{kh}(p) - \frac{1-4v}{8(1-v)}\delta_{kh}\dot{\varepsilon}^P_{mm}(p) \quad \text{(plane strain)}\end{aligned} \tag{5}$$

2.2. Constitutive equations

Continuum creep damage constitutive equations can be expressed as follows, see, for example, in Refs. [2–7]:

$$\frac{\mathrm{d}\varepsilon^c_{ij}}{\mathrm{d}t} = \frac{3}{2}A\frac{(\sigma_{\mathrm{eq}})^{n-1}S_{ij}}{(1-\omega)^n}t^{(m-1)} \tag{6}$$

where A, m and n are material constants, σ_{eq} is the equivalent stress and S_{ij} is the deviatoric stress. The damage parameter, ω, is scalar quantity, varying from 0 (no damage) to 1 (complete failure) and its rate of change is given in terms of a rupture stress, σ_{r}, as follows:

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = M\frac{(\sigma_r)^{\chi}}{(1+\phi)(1-\omega)^{\phi}}t^{(m-1)} \tag{7}$$

where M, ϕ and χ are continuum damage material constants. The rupture stress, σ_{r}, is given as a function of maximum principal stress, $\tilde{\sigma}$, and the equivalent stress, σ_{eq}, as follows [8]:

$$\sigma_{\mathrm{r}} = (1-\alpha)\sigma_{\mathrm{eq}} + \alpha\tilde{\sigma} \tag{8}$$

where α is a material constant, ranging from 0 to 1 and its value can be determined from notched specimen uniaxial tests, see in Ref. [9]. Under creep conditions, the effect of local plastic deformations on the α-values can be neglected [10].

For a material obeying the von Mises yield criterion and linear isotropic hardening, the plastic strain increments are given by the following incremental elasto-plastic flow rule:

$$\dot{\varepsilon}^p_{ij} = \frac{3}{2}\left(\frac{\dot{S}_{kl}\dot{\varepsilon}_{kl}}{1+H'/3\bar{\mu}}\right)\frac{\dot{S}_{ij}}{(\dot{\sigma}_{\mathrm{eq}})^2} \tag{9}$$

in which $\dot{S}_{ij}$ and $\dot{\sigma}_{\mathrm{eq}}$ denote the current deviatoric stress tensor and the equivalent stress, respectively. $\dot{\varepsilon}_{kl}$ is the total strain increments, $\bar{\mu}$ is the shear modulus and H' is the slope of the equivalent stress–plastic strain curve of the material at the current equivalent stress, and is also known as the plastic modulus of the material. It is defined for isotropic hardening materials as follows:

$$H' = \frac{\mathrm{d}\sigma_{\mathrm{eq}}}{\mathrm{d}\varepsilon^p_{\mathrm{eq}}} - = \frac{\dot{\sigma}_{\mathrm{eq}}}{\dot{\varepsilon}^p_{\mathrm{eq}}} \tag{10}$$

The BE formulations for elasto-plastic problems are widely discussed in the literature (see, for example, Refs. [1,11–15]). For plastic analysis, in general, it is more practical to reduce the number of the load steps to a minimum and to attempt to find average values for the stress and strain increments which are reasonably representative of the particular load step. Although experience is needed to specify the optimum size for the load steps, it is possible to roughly determine the size of a particular load step by considering the maximum allowable deviation from proportional loading for the present load step [1,16,17].

It is clear from Eq. (8) that the rupture stress is a combination of the von-Mises equivalent stress and the maximum principal stress. As in some cases, e.g. in the weld specimens [10], the maximum principal stresses are higher than the von-Mises equivalent stresses, then the maximum principal stresses are most important for the damage failure life.