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A constitutive theory of particulate-reinforced viscoelastic materials with partially debonded microvoids

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Abstract

In particulate-reinforced viscoelastic materials, the interfacial debonding between particles and matrix is usually a significant damage mechanism. In this paper, a new theoretical model of constitutive relation for particulate-reinforced viscoelastic materials is proposed. In the new constitutive model, both the reinforced effect of particles and weak effect due to microdamage evolution are taken into account. First, the deformation of a nucleated microvoid due to partially interfacial debonding between a particle and infinite viscoelastic matrix is analyzed in terms of Eshelby's equivalent inclusion method. Secondly, the porosity of nucleated microvoids is calculated based on the computation of density function of nucleation rate, in which, the size distribution of particles is assumed to be a logarithmic normal function, and the debonding probability of interface between particles and matrix is assumed to obey Weibull's distribution. Thirdly, a new definition of average stress and average strain for nucleated microvoids is proposed with considering the effect of nucleation time. Based on definition, a new constitutive model is suggested. This new model is consistent with experimental results. Finally, the effects of loading rate, interfacial adhesive energy, geometrical parameters of particles, and relaxation time on the constitutive relation are examined numerically. The simulation results indicate that the increase of loading rate, volume fraction of particles, and relaxation time of matrix not only reinforces the stiffness of composites, but also speeds up the evolution of microdamage. Such a competition mechanism should be taken into account when designing particulate-reinforced composites.

Keywords: Constitutive relation; Damage evolution; Mesomechanics; Viscoelastic materials; Interfacial debonding

1. Introduction

One of the effective methods for improving the performance of mechanics of a matrix material is to fill particles within it. Various efforts have been devoted to the toughness, strength, stiffness, and dynamic failure behavior of such a matrix/particles system. For instance, Christensen and Lo analyzed effective shear properties of composites [1]. The three sphere and cylinder models proposed by them are very valid not only in the calculation of effective shear modulus, but also in the prediction of constitutive relation of composites. Nasser and Hori [2], and Mura [3] detailedly introduced application of theory of micromechanics in composites (including particulate-reinforced composites). Apart from the theory of micromechanics, numerical simulation in the mechanical behaviour of particulate-reinforced composites is also helpful method. For example, Saraev and Schmauder [4] studied the reinforced effect of particle in metal matrix composites (MMCs) by means of 3D FEM theory, and they found that if the particle volume fraction is kept constant, varying distribution of particles in space leads to considerable change in the overall mechanical behaviour of composites.

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In particulate-reinforced viscoelastic composite materials, such as the particle filled polymer, particles are usually applied to high stresses [5]. When the deformation of the material reaches to some threshold value, nucleation of microvoids will take place due to the interfacial debonding between particles and matrix [6,7]. Usually, the form of the interfacial debonding depends on stress triaxiality. The fully debonding of the interface may occur under the action of high stress triaxiality, and the nucleated microvoids are called as fully debonded voids. In such a situation, the debonded particle does not contact with matrix, and thus no longer be able to transfer stress [8]. Partially debonding of the interface, however, is caused by low stress triaxiality, and the nucleated microvoids in this case may be called as partially debonded voids.

Unlike a fully debonded void, a particle within a partially debonded void can transfer stress at the surface which contacts matrix. This implies that some stress components exist in the void. Otherwise, the growth of the void can be restricted by such a particle. Therefore, both the damage evolution and the macroscopic constitutive relation of the material are all different from those of the material under the action of high stress triaxiality.

Zhao and Weng have studied the constitutive relation of a two-phase composite with partially debonded inclusions [9]. The matrix material is assumed to be linear elastic, and the inclusions are divided into two types; one is a perfectly bonded particle and the other is a partially debonded particle. Since the partially debonded particle can only transfer part of stresses, it may be taken as an anisotropic inclusion. By means of Eshelby's equivalent inclusion method and Mori–Tanaka scheme, the effective modulus of the composite is predicted. Furthermore, they investigated the macroscopic constitutive relation of particulate-reinforced elastoplastic composite materials by using linear comparing material method as well as the effective stress definition of the second order moment [10]. The obtained constitutive relation is transverse isotropic due to the partially debonded void.

If the matrix material is a viscoelastic one, the growth of the nucleated microvoids depends on not only the remote strain history, but also the nucleation time [8]. Therefore, the macroscopic mechanical properties of the material should be influenced by the nucleation time of microvoids. Chen et al. discussed the constitutive relation of particulate-reinforced viscoelastic material with considering the evolution of fully deboned microvoids [8]. In literature, very few results were reported on the overall property of particulate-reinforced viscoelastic materials with considering the partially debonded voids.

In the present paper, the growth of nucleated microvoids is studied by virtue of the Eshelby's equivalent inclusion theory [11]. From theoretical analysis, we establish the relations among the microvoid strain, nucleation time,-and remote strain history. A general expression of void strain is derived.

In order to study the macroscopic constitutive relation of the material, a new integral expression is proposed to definite the average stress and average strain of nucleated microvoids. The expression of the constitutive relation is derived. Comparison of the present model with existing experimental results is also performed. The influences of the nucleation time, loading rate, viscosity of the matrix, interfacial adhesive energy, and parameters of particles on the overall mechanical properties of the material are examined numerically.

2. The strain of the partially debonded microvoid

The in situ tension test of the interfacial debonding between particles and polymeric matrix was carried out by Bai et al. [12]. The material of specimen is high density polyethylene filled with glass beads (HDPE/GB). The SEM results show that the microvoids due to partial debonding of interfaces may take place when the remote stress reaches to a critical value. In this section, the deformation of a partially debonded void is theoretically analyzed.

Using Stieltjes convolution of two time-related tensors $\mathbf{F}$ and $\mathbf{G}$ denoted by

$$\mathbf{F} * \mathrm{d}\mathbf{G} = \int_{-\infty}^{t} \mathbf{F}(t-\tau) : \dot{\mathbf{G}}(\tau)\mathrm{d}\tau. \tag{1}$$

the constitutive relation of the matrix material may be expressed as [13]

$$\boldsymbol{\sigma}_0 = \mathbf{L} * \mathrm{d}\boldsymbol{\varepsilon}_0, \quad \text{or} \quad \boldsymbol{\varepsilon}_0 = \mathbf{J} * \mathrm{d}\boldsymbol{\sigma}_0, \tag{2}$$

where $\mathbf{L}(t)$ and $\mathbf{J}(t)$ are the fourth order relaxation modulus and creep compliance, respectively. If the matrix material is isotropic and the Poisson's ratio v is assumed to be a constant, then $\mathbf{L}$ and $\mathbf{J}$ can be written as

$$\mathbf{L} = \widehat{\mathbf{L}}\, l(t), \quad \mathbf{J} = \hat{\mathbf{J}}\, j(t), \tag{3}$$

where $\widehat{\mathbf{L}} = \mathbf{L}(0), \hat{\mathbf{J}} = \mathbf{J}(0),\ \widehat{\mathbf{L}} = \hat{\mathbf{J}}^{-1},\ l(t)$ and $j(t)$ are the functions of time with $l(0) = j(0) = 1$.

Assume that the stress and strain in viscoelastic matrix material can be expressed by following product of time-varying and spatial-varying functions,

$$\begin{aligned} \boldsymbol{\sigma}_0(\mathbf{x},t) &= \hat{\boldsymbol{\sigma}}_0(\mathbf{x}) f(t), \\ \boldsymbol{\varepsilon}_0(\mathbf{x},t) &= \hat{\boldsymbol{\varepsilon}}_0(\mathbf{x}) g(t), \end{aligned} \tag{4}$$

where symbol "∧" denotes the spatial part. Substituting Eqs. (3) and (4) into Eq. (2), we obtain

$$f = l * \mathrm{d}g, \quad g = j * \mathrm{d}f, \tag{5}$$

and

$$\hat{\boldsymbol{\sigma}}_0(\mathbf{x}) = \widehat{\mathbf{L}} : \hat{\varepsilon}_0(\mathbf{x}), \quad \hat{\boldsymbol{\varepsilon}}_0(\mathbf{x}) = \hat{\mathbf{J}} : \hat{\sigma}_0(\mathbf{x}). \tag{6}$$

From Eq. (6) one can see that the relation between $\hat{\boldsymbol{\sigma}}_0$ and $\hat{\boldsymbol{\varepsilon}}_0$ is "linear elastic". Therefore, the Eshelby's equivalent inclusion method [11] may be employed to calculate the spatial part of the stress and strain of an elastic inclusion embedded in such a matrix material.