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Parameter identification for a TRIP model with backstress

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Abstract

The investigation of complex material behaviour of steel like transformation-induced plasticity (TRIP) is a large field of current research. The simulation of the material behaviour of work-pieces in complex situations requires a knowledge as deep as possible about phenomena like TRIP. In addition, there are effects in the case of non-constant stress which cannot be explained by the widely used Leblond model. Therefore, we consider a TRIP model taking into account backstress due to TRIP itself. Assuming a linear dependence of this backstress from the TRIP strain we derive formulas for calculation of the new TRIP parameters. Based on experimental data we calculate these parameters.

Keywords: Steel; Transformation-induced plasticity; Backstress; Testing strategies; Numerical calculations

1. Introduction and motivation

Phase transformations in steel under non-zero deviatoric stress yield a permanent volume-preserving deformation, even if the von Mises (macro) stress does not reach the yield stress. This phenomenon is called transformation-induced plasticity (TRIP) and cannot be described by classical plasticity. In the framework of small deformations TRIP can be taken into account by an additional (linearized) strain tensor $\varepsilon_{\mathrm{TRIP}}$. Here we consider the case of the formation of one phase (pearlite from austenite, e.g.) The corresponding well-known Franitza–Mitter–Leblond ansatz (cf. [1–6,9,10,13,17,18], e.g.) in its incremental form is

$$\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon_{\mathrm{TRIP}}(x,t) = \frac{3}{2}\kappa(\theta(x,t))\sigma^*(x,t)\frac{\mathrm{d}}{\mathrm{d}t}\Phi(p(x,t)), \tag{1}$$

where θ—temperature, σ^*—deviator of the stress tensor σ, $\kappa > 0$—Greenwood–Johnson parameter possibly depending on θ, Φ—saturation function, p—volume fraction of the forming phase, x—material point in the reference configuration, $t \geqslant 0$—time. Further, Φ is assumed to be a continuously differentiable function on $[0,1]$ with

$$0 < \Phi(p) < 1 \quad \text{for all } 0 < p < 1,\ \Phi(0) = 0,\ \Phi(1) = 1. \tag{2}$$

There are several proposals for Φ (cf. [6,17]), partially based on experiments, partially derived from theoretical considerations (cf. [9,12]). The software SYSWELD® [11] uses (1) and Φ as proposed in [9]. A suggestion for Φ without $\Phi(0) = 0$ can be found in [12]. Additionally, in accordance with experiments (cf. (20)), Φ must be monotone.

The model (1) works well in experimental situations with constant load. But there are experiments showing that it cannot describe the effect after unloading to zero when the TRIP strain decreases: The model (1) predicts a non-zero rest of the TRIP strain after unloading to zero (cf. [2,14,16], e.g.). In order to fill this gap, some authors introduce a backstress due to TRIP (cf. [7,14,15]), which leads to

$$\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon_{\mathrm{TRIP}}(x,t) = \frac{3}{2}\kappa_{\mathrm{b}}(\theta(x,t))(\sigma^*(x,t) - X^*_{\mathrm{TRIP}}(x,t)) \times \frac{\mathrm{d}}{\mathrm{d}t}\Phi_{\mathrm{b}}(p(x,t)), \tag{3}$$

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where X_{TRIP} is the backstress tensor due to TRIP. Since κ_{b} and Φ_{b} in (3) generally differ from the corresponding values in (1), we distinguish them. Let Φ_{b} also fulfil (2), (cf. (17)). The philosophy is as follows: For constant temperature and load the two models (1) and (3) describe TRIP correctly. But in the case of varying load the model (3) shows its advantage. A possible simple proposal for X_{TRIP} (in the absence of classical plasticity which can influence TRIP, cf. [15]) is

$$X_{\mathrm{TRIP}} = c\varepsilon_{\mathrm{TRIP}}, \tag{4}$$

where c is a further positive material parameter (cf. Chap. 4). Taking the volume preservation of TRIP into account, we get from (3) and (4) the following linear differential equation for $\varepsilon_{\mathrm{TRIP}}$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon_{\mathrm{TRIP}}(t) = &-\frac{3}{2}\kappa_{\mathrm{b}}(\theta(t))c\frac{\mathrm{d}}{\mathrm{d}t}\Phi_{\mathrm{b}}(p(t))\varepsilon_{\mathrm{TRIP}}(t)\\ &+\frac{3}{2}\kappa_{\mathrm{b}}(\theta(t))\sigma^*(t)\frac{\mathrm{d}}{\mathrm{d}t}\Phi_{\mathrm{b}}(p(t)),\end{aligned} \tag{5}$$

where we oppress the variable x. The unique solution of (5) corresponding to the initial condition $\varepsilon_{\mathrm{TRIP}}(0) = 0$ at the beginning $t = 0$ describes the TRIP strain as a function of material parameters, temperature and stress

$$\begin{aligned}\varepsilon_{\mathrm{TRIP}}(t) = &\frac{3}{2}\int_0^t \kappa_{\mathrm{b}}\sigma^*\frac{\mathrm{d}}{\mathrm{d}t}\Phi_{\mathrm{b}}(p(s))\\ &\times \exp\left(-\frac{3}{2}\int_s^t c\kappa_{\mathrm{b}}\frac{\mathrm{d}}{\mathrm{d}t}\Phi_{\mathrm{b}}(p(\tau))\mathrm{d}\tau\right)\mathrm{d}s.\end{aligned} \tag{6}$$

We note that c may depend on the temperature and on other entities. The relation (6) is the starting point for testing the model and determining the material parameters through experimental data.

2. Uniaxial tension–pression tests with small specimen-constant load

Now we want to derive special relations in order to determine the material parameters from experimental data. For this reason we consider a small cylindrical probe of length l with circular or annulus-like cross-section of (outer) diameter d. Along the axis we exert a stress F sufficiently smaller than the yield stress of the weaker phase (here: austenite). Therefore there is no classical plasticity. We position a rectangular coordinate system such that the probe's axis lies on the x_1-axis. In this setting the deviator σ^* of the stress tensor σ reads as

$$\sigma^*_{11} = \frac{2}{3}F, \quad \sigma^*_{22} = -\frac{1}{3}F, \quad \sigma^*_{33} = -\frac{1}{3}F, \quad \sigma^*_{ij} = 0, \quad \text{for } i \neq j. \tag{7}$$

(For tension we have $F > 0$, for compression $F < 0$. The following works in both cases.) We assume that the specimen are of spatially uniform temperature $\theta(t)$, that $\theta(t)$ and the load $F(t)$ can be controlled and that the actual length $l(t)$ and the outer diameter $d(t)$ can be measured at all times $t \geqslant 0$. As in [17,18] we get for the bulk strain (cf. (6), (7))

$$\begin{aligned}\varepsilon_{\mathrm{bulk},11} = \frac{l(t)-l}{l} = &\frac{1}{E(\theta(t))}F(t) + \frac{1}{3}\frac{\rho_0 - \rho(\theta(t))}{\rho(\theta(t))}\\ &+ \int_0^t \kappa_{\mathrm{b}}(\theta(s))F(s)\frac{\mathrm{d}}{\mathrm{d}t}\Phi_{\mathrm{b}}(p(s))\\ &\cdot \exp\left(-\frac{3}{2}\int_s^t c\kappa_{\mathrm{b}}(\theta(\tau))\frac{\mathrm{d}}{\mathrm{d}t}\Phi_{\mathrm{b}}(p(\tau))\mathrm{d}\tau\right)\mathrm{d}s,\end{aligned} \tag{8}$$

$$\begin{aligned}\varepsilon_{\mathrm{bulk},22} = \frac{d(t)-d}{d} = &\frac{-\nu(\theta(t))}{E(\theta(t))}F(t) + \frac{1}{3}\frac{\rho_0 - \rho(\theta(t))}{\rho(\theta(t))}\\ &- \frac{1}{2}\int_0^t \kappa_{\mathrm{b}}(\theta(s))F(s)\frac{\mathrm{d}}{\mathrm{d}t}\Phi_{\mathrm{b}}(p(s))\\ &\cdot \exp\left(-\frac{3}{2}\int_s^t c\kappa_{\mathrm{b}}(\theta(\tau))\frac{\mathrm{d}}{\mathrm{d}t}\Phi_{\mathrm{b}}(p(\tau))\mathrm{d}\tau\right)\mathrm{d}s,\end{aligned} \tag{9}$$

where $\varepsilon_{\mathrm{bulk}}$—the linearized bulk strain tensor, ρ_0—the density of the (stress free) reference configuration (with respect to the start temperature θ_0), $\rho(\theta)$—the actual density, E, ν—Young modulus and Poisson ratio (generally being phase dependent). For abbreviation we set

$$\varepsilon_L := \frac{l(t)-l}{l}, \quad \varepsilon_D := \frac{d(t)-d}{d}, \quad \gamma_F(t) := \frac{1-2\nu(\theta(t))}{E(\theta(t))}F(t). \tag{10}$$

The Eqs. (8) and (9) yield

$$\varepsilon_L(t) + 2\varepsilon_D(t) = \gamma_F(t) + \frac{\rho_0 - \rho(\theta(t))}{\rho(\theta(t))}, \tag{11}$$

$$\begin{aligned}\varepsilon_L(t) - \varepsilon_D(t) = &\frac{1+\nu(\theta(t))}{E(\theta(t))}F(t) + \frac{3}{2}\int_0^t \kappa_{\mathrm{b}}(\theta(s))F(s)\\ &\times \frac{\mathrm{d}}{\mathrm{d}t}\Phi_{\mathrm{b}}(p(s))\\ &\cdot \exp\left(-\frac{3}{2}\int_s^t c\kappa_{\mathrm{b}}(\theta(\tau))\frac{\mathrm{d}}{\mathrm{d}t}\Phi_{\mathrm{b}}(p(\tau))\mathrm{d}\tau\right)\mathrm{d}s.\end{aligned} \tag{12}$$

These last two equations allow to investigate the effects of TRIP and stress-dependent phase transformation separately. Via (11) the evolution of the phase fraction p can be determined based on the measured data (cf. [17,18]) for details. Furthermore, let's derive a formula for the phase fraction of the forming phase in a two-phase system with complete transformation in the case of constant transformation temperature, i.e.,

$$\theta(t) = \theta_0 = \text{const.} \tag{13}$$

Letting t_∞ be a sufficiently large time after which the transformation can be assumed as completed, i.e., $p(t_\infty) = 1$, we obtain from (11) (cf. [17,18])

$$p(t) = \frac{\varepsilon_L(t) + 2\varepsilon_D(t) - \gamma_F(t)}{\varepsilon_L(t_\infty) + 2\varepsilon_D(t_\infty) - \gamma_F(t_\infty)}, \tag{14}$$

or, neglecting the elastic part, its occasionally used approximation (cf.[1–4])

$$p(t) = \frac{\varepsilon_L(t) + 2\varepsilon_D(t)}{\varepsilon_L(t_\infty) + 2\varepsilon_D(t_\infty)}. \tag{15}$$