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Network model of fluid flow in semi-solid aluminum alloys

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Abstract

A network model is presented to simulate solidification and fluid flow within a semi-solid aluminum alloy. The model consists of a set of connected channels representing the interdendritic liquid and its possible flow directions at high solid fractions (>0.7). The individual channels react according to solidification rules upon liquid solute convection and changes of the local temperature. This dynamic network model has been designed in such a way that a qualitative study of the interaction between the channels is possible. The simulations of the alloys considered show good agreement with the expected macroscopic fluid flow behavior. The simulations indicate a possible reason for deviations from the Kozeny–Carman relationship in measurements of semi-solid alloys.

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1. Introduction

Early simulations of the solidification of alloys often used an averaging approach [1]. This approach describes all physical properties within a representative volume element (REV) in terms of fraction solid or liquid. Even though the approach has had many refinements and extensions [2–6] its principles are still used today to simulate solidification on a macroscopic level (e.g. [5]). Since the early simulations however the uncertainty remains of how to determine the fluid flow rate within a semi-solid REV given the pressure on its boundary. With few exceptions averaged based simulations have assumed that Darcy's law is applicable. According to Darcy's law the fluid flow rate is proportional to the applied pressure difference [7].

The problem which arises is to find an adequate proportionality constant, i.e. to determine permeability. An obvious way to do this is to measure it using an alloy or a similar substance. This has been done many times with various experimental settings e.g. [8–12]. Apart from the obvious difficulty that the underlying micro-structure of the semi-solid might vary during the experiment, a number of additional limitations of the experiments have been described [19]. Furthermore, the measurements show a rather wide spread, which makes it difficult to fit them to a general rule relating permeability to the fraction of solid (or liquid) [20,21]. However, most experiments report however good agreement with the Kozeny–Carman relationship (1), at least within a certain range of liquid or solid fractions (about $0.5 < f_s < 0.9$ in [19]) and with the right (individual) fitting parameters.

The Kozeny–Carman relationship is given by

$$K = \frac{f_l^3}{k_c S_S^2}, \tag{1}$$

where K is the permeability, f_l is the volume fraction of liquid, S_S is the solid–liquid interfacial area per unit volume and k_c is the Kozeny constant. This equation can directly be derived from the assumption that the whole

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porous and semi-solid alloy (or other porous media) can be represented by a set of straight and parallel tubes. This rather crude approximation shows, for many sorts of porous media, a remarkably good agreement with reality as long as the Kozeny constant can be estimated [22–24].

The Kozeny constant can be expected to vary generally most at high and very low liquid fractions [25]. For very low liquid fractions the known experiments using semi-solid alloys seem not to be very suitable. Therefore, it is not surprising that fluid flow has been numerically simulated using micro-photographs of the structure of quenched alloys [26–31]. This technique ignores completely the dynamic changes of the micro-structure and there are some uncertainties on how the quenching process affects the micro-structure. Until now, there remains some doubts about the applicability of the Kozeny–Carman relationship and the correct value of the Kozeny constant [32].

Another approach to describe permeability was adapted by Wang and Beckermann [33–37]. The main idea behind the derived equation is that the dendritic micro-structure of the semi-solid can be represented by a set of spheres. Through the distinction of intra- and inter-spheric liquid, detailed description of fluid flow and solidification becomes possible. New measurements show that the expression established by these authors and the Kozeny–Carman relationship do not deviate significantly over a wide range of liquid fractions [12].

The fluid flow within solidifying alloys has been studied in various other works. Among them, theoretical considerations including closure problems are used in [13–15]. More detailed (micro-)structures were simulated applying the phase-field method [16–18]. None of them however made any conclusive remarks about the usability of the Kozeny–Carman relationship.

For this purpose a cellular automata model which seems to have been restricted to a small 3-D region around a grain was used by Brown et al. [32]. The major contribution of their work is that the authors proposed a new value for the Kozeny constant (i.e. 1 instead of 5) to allow for a better agreement of the Kozeny–Carman relationship with their numerical study. It remains however unclear whether this deviation from the original constant is due to the restricted size of the simulation.

In this article the well established idea of a network model [38–43] is extended and newly applied in the field of alloy solidification. The main idea is to represent fluid flow conducts within the semi-solid by a set of geometrical simplified and interconnected channels. Solidification is then included on an individual channel basis, while fluid flow through the represented semi-solid is determined by the flow through the whole channel network.

The advantage of this kind of model is that it allows for investigations which are experimentally not possible and which involves interactions between a lot (i.e. hundredths) of grains at high solid volume fractions. Therefore, the model is suited to investigate the Kozeny–Carman relationship. Furthermore, it is expected that percolation or self induced macro-segregation effects will be revealed with this approach if they exist within solidifying alloys.

2. Model description

The semi-solid state of alloys often consists of solidifying grains within liquid. From a numerical point of view, there are two main approaches to study such semi-solid alloys. On the one hand, it is possible to study development of a few individual grains in detail. Because this involves accurate simulation on the scale of microns and less, the resulting models are called microscopic models. On the other hand, it is possible to average over a large number of grains and study the macroscopic behavior of the semi-solid. With averaging, all conservation equations will be written in terms of liquid or solid volume fractions. Note that the conservation equations are not directly related to the actual microscopic structure.

There exists various ways of coupling microscopic and macroscopic models. But the coupling usually involves assumptions (e.g. restriction to 2-D, no shrinkage) which considerably reduce the complexity of the model. Therefore, all micro–macro models are somehow restricted like the model described below. It relies heavily on a number of assumptions as stated below. However, the model restrictions are chosen in such a way that the interactions of the macroscopic liquid flow and the individual behavior of the grains can be retained. For this reason, this model has the characteristics of a mesoscopic model.

2.1. Basic concept

The basic idea behind the simulation model presented in this article is that the complex structure of the mush, i.e. of the dendritic grains surrounded by the liquid phase, can be represented by a set of interconnected liquid channels. In this paper 2-D networks have been chosen to represent the mush. As shown in Fig. 1 the channel network resembles a distorted padding of a regular hexagonal network.

To create the channel networks a random distribution of grain centers is used. The Voronoi diagram which belongs

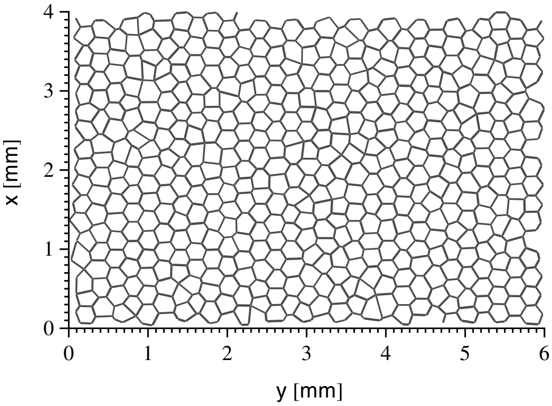

Fig. 1. Liquid channel network.