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Finite element analysis of crack perpendicular to bi-material interface: Case of couple ceramic–metal

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Abstract

The finite element method is used to analyse the interaction effect between a crack and an interface in a ceramic/metal bi-material. The global approach is based on the energy release rate used as criteria of propagation of the crack. The effects of the elastic properties of two bonded materials were highlighted in order to evaluate the conditions for the crack deflection by the interface as well as the effects of the thickness of the bi-material assembly and the distance between the crack tip and the interface.

Keywords: Energy release rate; Bi-material; Interface; Crack penetration; Crack deflection

1. Introduction

A perpendicular crack to a bi-material interface has attracted the attention of many investigators. Zak and Williams [1] used the eigenfunction expansion method to analyse the stress singularity ahead of a crack tip, which is perpendicular and terminating at the Interface. Cook and Erdogan [2] used the Mellin transform method to derive the governing equation of a finite crack perpendicular to the interface and obtained the stress intensity factors. Erdogan and Biricikoglu [3] solved the problem of two bounded half planes with a crack going through the interface. Bogy [4] investigated the stress singularity of an infinite crack terminated at the interface with an arbitrary angle. Wang and Chen [5] used photoelasticity to determine the stress distribution and the stress intensity factors of a crack perpendicular to the interface. Lin and Mar [6], Ahmad [7] and Meguid et al. [8] used finite element to analyze cracks perpendicular to bi-material in finite elastic body. Chen [9] used the body force method to determine the stress intensity factors for a normal crack *d* terminated at a bi-material interface. Wang and Stahle [10] investigated a crack growing towards a bi-material interface. Their results showed that the crack can be deflected and to follow a smooth curved path. Singly and doubly deflected interface cracks were considered within the limitations of plane strain. He and Hutchinson [11] also considered cracks approaching the interface at oblique angles. Gupta et al. [12] extended He and Hutchinson's work [11] to the area of anisotropic materials for the case of a crack approaching perpendicular to the interface. Martinez and Gupta [13] also examined the effect of anisotropy on the crack deflection by manipulating the anisotropy-related parameters including the other Dundurs' parameter. In this study the finite element methods is used to calculate the strain energy release rates of ceramic–metal composite cracking. The first part of this work has been contributed to the crack normal to the interface and the second one is consecrated to the deflection/penetration of the crack. The effects of the distance between the crack tip and the interface were highlighted

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as well as the effects of the elastic properties of two bonded materials.

2. Crack behavior at interface

For the plane problem of a bi-material plate with a crack normal at the interface, the stresses near the crack tip that touches the interface are found to be of the following from [9]:

$$\sigma_{ij} = \frac{K}{r^{1-\lambda}} f_{ij}(\theta) \quad (1)$$

where r and θ are, respectively, the radial distance and the polar angle; K is the stress intensity factor; $f_{ij}(\theta)$ represents the angular distribution of the singular stress field.

Lin and Mar [6] gave an explicit expression for $f_{ij}(\theta)$. The values of $f_{ij}(\theta)$ for special material combination were also calculated numerically by Cook and Erdogan [2] and Ahmed [7].

λ defines the strength of stress singularity. It has been shown that the value of λ, which is real ($0 < \lambda < 1$) and a function of the elastic properties of the present materials, λ can be determined as a root of the following characteristic equation [1]

$$2\lambda^2(\alpha-\beta)(\beta+1) - \alpha + \beta^2 + (1-\beta^2)\cos(\pi\lambda) = 0 \quad (2)$$

For identical elastic properties $\lambda = 1/2$, but for material 1 stiffer than the material 2, $\lambda < 1/2$, while for material 2 stiffer than the material 1, $\lambda > 1/2$.

α and β are the Dundurs bi-material parameters [14], which are defined as

$$\alpha = \frac{E'_1 - E'_2}{E'_1 + E'_2} \quad \text{and} \quad \beta = \frac{\mu_{1(1-2\nu_2)} - \mu_{2(1-2\nu_1)}}{\mu_{1(1-\nu_2)} + \mu_{2(1-\nu_1)}}, \quad j = 1, 2$$

where E is the Young modulus, $E'_j = E_j$ for plane stress, $E'_j = E_j/(1-\nu_j)$ for plane strain, (1 and 2 denote the material), μ and ν are, respectively, the shear modulus and the Poisson ratio.

The parameter α approaches +1 when the stiffness of material 1 is extremely large compared to the stiffness material 2, and both parameters become zero in the case of homogeneous material systems. If the two materials 1 and 2 are switched both α and β change signs. If a crack reaches an interface, there are at least three possible crack paths for the crack. Fig. 1 shows the simplest possible failure paths in an bi-material model: (a) crack penetration across the interface; (b) crack deflection on one side of the interface (singly deflected crack); and (c) crack deflection on both sides (doubly deflected crack).

For a crack perpendicular to the interface and under applied load parallel to the interface, the strain energy release rates as a function of crack extension a_d along

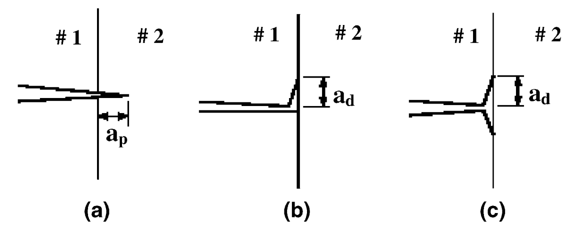

Fig. 1. Crack penetration or deviation on the interface. (a) Penetration, (b) single deviation and (c) double deviation.

the interface (deflection) and a_p in to the interface (penetration) are well-known to be of the forms [11,15]

$$G_d = \frac{\left[\frac{1-\nu_1}{\mu_1} + \frac{1-\nu_1}{\mu_2}\right]}{4\cosh^2\pi\varepsilon}(K_1^2 + K_2^2) \quad (3)$$

$$G_p = \frac{1-\nu_2}{2\mu_2} K_p^2 \quad (4)$$

In Eqs. (3) and (4), G_d and G_p are, respectively, the strain energy release rate for deflection and penetration; K_1 and K_2 are the stress intensity factors for the interface crack; K_p is the stress intensity factor for the case of penetration, ε is a function of material constants

$$\varepsilon = \frac{1}{2\pi} \ln\left(\frac{1-\beta}{1+\beta}\right)$$

The condition for crack deflection can be expressed as follows [11,13]:

$$\frac{G(\psi)}{G_2^c} < \frac{G_d}{G_p} \quad (5)$$

where $G(\psi)$ and G_2^c are the toughness of the interface and the toughness of the penetrated material (2).

3. Results and discussion

3.1. Crack perpendicular to the interface

To study the interaction effect of a crack with an interface, let consider a plate formed by a ceramic/metal bi-material (Fig. 2). A perpendicularly oriented crack to the interface is localized in the ceramic. Under the effect of the applied stress σ, this crack is susceptible to propagate until the interface. The plate is modeled by eight node isoparametric quadratic elements (Fig. 3). The singularity in crack tip is modeled by special elements of 1/4 point type [16].

Table 1 regroups the two Dundurs parameters α and β and the Young moduli ratios E_1/E_2 for different couples considered in this study.

The obtained results are presented in Fig. 4. This one illustrates the variation of the energy release rate G_1/G_0