Chemical Engineering Science 66 (2011) 766–776

Contents lists available at ScienceDirect

Chemical Engineering Science

journal homepage: www.elsevier.com/locate/ces

A theoretical model for droplet breakup in turbulent dispersions

Luchang Han, He'an Luo*, Yuejin Liu

College of Chemical Engineering, Xiangtan University, Hunan 411105, PR China

ARTICLE INFO

Article history:
Received 17 February 2010
Received in revised form
23 November 2010
Accepted 25 November 2010
Available online 7 December 2010

Keywords:
Droplet
Surface oscillation
Energy constraint
Breakup
Daughter size distribution
Turbulent dispersion

ABSTRACT

A theoretical model for the prediction of droplet breakup rate and daughter size distribution in turbulent dispersions has been developed. It considers the breakup contributed by a novel breakup criterion based on surface energy density increase, the droplet surface oscillation from previous collision and the eddies larger than original droplets. The breakup rate and daughter size distribution predicted by this model show a good agreement with the experimental data reported recently.

1. Introduction

In turbulent dispersions, the breakup of fluid particles usually determines the size distribution of dispersed phase and the interfacial area, and thus plays a crucial role in mass transfer, heat transfer and chemical reaction. The population balance model (PBM) can predict such size distribution and interfacial area, provided that breakup rate and daughter size distribution are given. Many studies have shown that the prediction precision of PBM strongly depends on both of them (Chen et al., 2005; Wang et al., 2006; Patruno et al., 2009), so it is needed to investigate the breakup of fluid particles in depth.

In past decades, many studies have focused on the fluid particles breakup in turbulent flows, a number of mathematical models have been presented, Lasheras et al. (2002) and Liao and Lucas (2009) have made excellent and comprehensive literature reviews. These models can be classified as two groups. The first is semi-empirical or semi-theoretical model, mainly based on a deterministic or semi-theoretical method to obtain the breakup rate and presume a matching daughter size distribution function, for example, a uniform or normal distribution (Lasheras et al., 2002; Liao and Lucas, 2009). Such model usually contained several unknown parameters that need to be determined by experiment. The second is called the phenomenological model, mainly based on the probability theory and the gas molecule collision dynamics, such model generally do not contain unknown parameters to be determined experimentally, and the daughter size distribution function can be derived from the breakup rate directly, such as the models of Luo and Svendsen (1996), Lehr and Mewes (2001), Lehr et al. (2002), Wang et al. (2003), and Zhao and Ge (2007).

In the literature, many breakup criteria have been proposed (please see the details reviewed by Lasheras et al., 2002; Kostoglou and Karabelas, 2005; Liao and Lucas, 2009). In recent years, the surface energy increase proposed by Luo and Svendsen (1996) has been widely used by many authors. They defined the critical turbulent eddy kinetic energy as the surface energy increase resulting from the breakage, that is, $e(\lambda) \geq c_f \pi d_0^2 \sigma$, here, $c_f = f_v^{2/3} + (1-f_v)^{2/3} - 1$, f_v is the breakup volume fraction and d_0 is the original diameter of particle. However, Hagesaether et al. (2002) and Wang et al. (2003) have pointed out that the model determined by this criterion has a shortcoming, i.e., as $f_v \to 0$ or 1, the required critical turbulent eddy kinetic energy tends to zero, which means that an eddy of any size can break the particle. Kostoglou and Karabelas (2005) thought that the derivation of Luo and Svendsen (1996) is incorrectly based on the concept of conditional breakage probability for a given fragment size f_v, overlooking that eddies incapable of producing fragments of size f_v can be efficient in producing fragments of smaller size. Under the premise of the surface energy increase, they proposed improved models, respectively. Hagesaether et al. (2002) presented a surface energy density criterion, i.e., turbulent eddy kinetic energy per unit volume must be greater than or equal to the surface energy per unit volume of smaller daughter particle, $\rho_c u_\lambda^2/2 \geq 6\sigma/d_1$, which was used to determine the diameter of smaller daughter particle, d_1. Wang et al. (2003) used a combination of the surface energy increase and the capillary pressure ($\rho_c u_\lambda^2/2 \geq \sigma/d_1$). They thought that the surface energy increase determined the maximum breakup fraction, $f_{v,max}$, and the capillary pressure determined the minimum breakup

* Corresponding author.
E-mail address: hluo@xtu.edu.cn (H. Luo).

0009-2509/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ces.2010.11.041

fraction, $f_{v,min}$, they assumed that the breakup probability density in the range $f_{v,min} \le f_v \le f_{v,max}$ obeys a uniform distribution. Kostoglou and Karabelas (2005) presented a unified framework for developing the breakage functions and showed that existing models can be derived in a systematic and consistent way. We notice that the criteria of the energy density and the capillary pressure have a similar final type, i.e. $\rho_c u_\lambda^2/2 \ge c\sigma/d_i$, where c is a coefficient. For convenience, they are all called dynamic pressure criterion (or constraint) in this work. It is worth to be noted that the turbulent velocity of eddy, u_λ, could be viewed as the volume averaged velocity, thus the eddy kinetic energy, $e(\lambda)$, could be represented by $\rho_c u_\lambda^2/2\pi\lambda^3/6$. Then the corresponding critical eddy kinetic energy determined by the dynamic pressure criterion could be written as $c\sigma/d_i\pi\lambda^3/6$. It is likely that $c\sigma/d_i\pi\lambda^3/6$ is smaller than $c_f\pi d_0^2\sigma$.

Recently, Andersson and Andersson (2006b) and Zhao and Ge (2007) proposed a more general combination of the surface energy increase and the dynamic pressure, i.e., $e(\lambda) \ge \max(c_f\pi d_0^2\sigma, 2\sigma/d_i\pi\lambda^3/6)$, where i denotes 0 (Andersson and Andersson, 2006b) or 1 (Zhao and Ge, 2007). Andersson and Andersson (2006b) introduced a concept of available turbulent eddy kinetic energy to the required eddy energy, and the coefficient, c_f, was taken as a constant 0.3 according to their experiment. They considered the contribution of turbulent eddies larger than fluid particle to the overall breakup rate. An interaction frequency that does not need to know the relative velocity was proposed in their model, and an additional model of daughter size distribution is needed. Zhao and Ge (2007) considered the eddy energy efficiency. Unlike other models, the breakup fraction was defined as the ratio of turbulent eddy volume to parent particle volume in their model. It can reduce the original double integration to an integral type. Nevertheless, Liao and Lucas (2009) have pointed out that this definition is still open to validation. It may become incomprehensible in the case of eddies larger than fluid particle since the breakup fraction will exceed one (i.e., $f_v = \lambda/d_0 > 1$).

Several models purely from a pressure point of view can be found in the literature. Martínez-Bazán et al. (1999a) considered the contribution of the average turbulent stress at a distance of d_0 to the breakup, and proposed that as the average deformation force produced by the turbulent fluctuation is greater than or equal to the surface energy per unit parent particle volume, fluid particle will break, i.e., $\rho_c \overline{\Delta u^2}(d_0)/2 \ge 6\sigma/d_0$. Through this criterion, a critical breakup diameter, d_c, can be obtained. Lehr et al. (2001, 2002) proposed a dynamic pressure criterion, i.e., $\rho_c u_\lambda^2/2 \ge c\sigma/d_1$, the constant, c is 1 (Lehr and Mewes, 2001) or 2 (Lehr et al., 2002). They thought that only the turbulent eddies of size between d_1 and d_0 can cause the breakup of fluid particle.

As mentioned above, these breakup criteria in previous models can be regarded that they are associated with the eddy energy since the critical dynamic pressure can be transformed into the form of required critical eddy energy, although the presentations of these criteria are different. It is not difficult to make two interesting points from the criteria of $c\sigma/d_1$ and $c\sigma/d_0$ under the condition of an eddy of given size, namely: (i) when forming a daughter particle of given size from parent particles of different sizes, the critical eddy energies are the same for the criterion of $c\sigma/d_1$; (ii) when forming daughter particles of different sizes from a given parent particle, the required critical energies are the same for the criterion of $c\sigma/d_0$. The two points appear not easy to be understood because in the case (i) the surface energy increase resulting from the breakup has indicated that the required critical eddy energy increases with the increasing diameter of parent particle. However, it should be noted that the overall breakup rate increases with the increasing parent particle diameter due to a wider size range of eddies contributing to the breakup. It is clear that the critical eddy energies for breaking different daughter particles from a given parent particle are not the same. As pointed out by Hagesaether et al. (2002), the breakup is likely controlled by dynamic pressure criterion (it could also be seen from $(c\sigma/d_i\pi\lambda^3/6) \ge c_f\pi d_0^2\sigma$ mentioned above) in the range of the most, even the whole breakup fractions. In this case, the existing dynamic pressure criteria appear unable to account for the breakup result fully. It is worth to be noted that the breakup rate and daughter size distribution are very sensitive to d_i/c, which was often referred to as an estimated value of the so-called minimum of curvature radius of the deformed particle, R_{min} in the literature ($R_{min} \approx d_1$, Lehr and Mewes, 2001; Wang et al., 2003; $R_{min} \approx d_1/2$, Lehr et al., 2002; Zhao and Ge, 2007; $R_{min} \approx d_0/2$, Andersson and Andersson, 2006b). It implies that the possible R_{min} needs to be further and carefully investigated, for example, considering the actual curvature radius varying with the time and the location during the deformation. In addition, it may also be advisable to avoid the determination of R_{min}.

In the literature, several recent experiments have focused on the daughter size distributions of bubbles and droplets (Galinat et al., 2005; Podgórska, 2006; Andersson and Andersson, 2006a; Tcholakova et al., 2007; Zaccone et al., 2007). In Andersson and Andersson's experiment, an obvious difference between bubbles and droplets breakup has been observed, that is, bubbles often break into two unequal-sized fragments and its probability is the highest, whereas low viscous droplets often break into two main equal-size daughter droplets and several satellite droplets, Tcholakova et al. (2007) and Zaccone et al. (2007) also observed the similar tendency for droplets breakup. Andersson and Andersson (2006a) further pointed out that although multiple breakage looks like the most frequent outcome, binary breakup may still be a reasonable assumption, if the volume and the interfacial area of the residual fragments are negligible. In addition, Andersson and Andersson (2006a) proposed an internal flow redistribution mechanism to explain the unequal-sized breakage for bubbles; they thought that this mechanism can accelerate the internal flow from the high pressure region to the low pressure region due to the pressure difference between the two ends of the deformed bubble. Whereas it cannot occur for droplet since the density difference between droplet and surrounding fluid is too small to produce such flow redistribution. This mechanism appears physically reasonable.

As seen from the above review, previous breakup models show obviously different characteristics and the comparisons of the predictions by them with the available experimental data are still needed. Furthermore, the contributions of the eddies larger than the droplets and the surface oscillation from previous collision to overall breakup rate were often neglected without validation in the literature. However, a recent experiment reported that droplets are subject to large scale deformations, i.e., close in size to and up to three times larger than the droplet, prior to breakup, and the surface oscillation is obvious. It means that the larger eddies and the surface oscillation need to be considered (Andersson and Andersson, 2006a). Thereby it will be valuable to develop a theoretical model considering these effects for the droplet breakup.

2. Model development

In turbulent dispersions, the breakup processes of fluid particles are very complex. Therefore, certain simplifications are necessary.

(1) It is commonly believed that more than one mechanism for droplet breakage may exist in turbulent dispersions, since a droplet is not only exposed to a turbulent field, but is also subjected to both inertial and viscous forces (Luo and Svendsen, 1996). In the present work, we focus on the low viscous and small droplets, thus the effect of viscous shear and inertial forces during the deformation can be neglected (Andersson and Andersson, 2006a). That is, the breakup is mainly caused by the turbulence.