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Analysis of pulsatile flow and its role on particle removal from surfaces

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ABSTRACT

The use of pulsatile flow for energy efficient particle removal from surfaces is evaluated through modeling calculations. The governing equation for pulsatile flow in a channel between parallel plates with an oscillatory pressure input is solved and wall shear stress, identified as a measure for particle removal, calculated for fixed power input. It is observed that as the frequency of oscillation is increased the average wall shear stress with an oscillatory pressure input is higher than the corresponding steady state value only above a critical frequency. Similar results are obtained for pulsatile flow in a pipe. Explanation for this observation is presented based on how velocity profile changes as a function of frequency and consequently its effect on wall shear stress versus power dissipated. Based on these observations we propose that there is a critical frequency above which an oscillatory pressure input will be energy efficient for particle removal.

1. Introduction

Historically, pulsatile flow of a liquid through a tube has been studied for its relevance in biological flows beginning with the seminal work of Womersley (1955). Edwards and Wilkinson (1971) considered the application of pulsatile flow to address industrially relevant questions such as how to achieve a higher flow rate for a given energy input or how to achieve a higher mass transfer coefficient for a given flow rate. Fan and Chao (1965) calculated the excess power dissipated due to harmonic motion superimposed on a steady flow and observed that the excess power decreases with an increase in oscillatory frequency. Recently pulsatile flow has also been evaluated for enhanced cleaning of fouling layers (Bode et al., 2007; Celnik et al., 2006; Gillham et al. 2000), and separation of species (Thomas and Narayanan, 2001). Qi et al. (2008) have modeled pulsatile flow in rectangular channels with application towards cleaning of micro-fluidic devices. The context in which we consider pulsatile flow is in its application towards energy efficient removal of particles and fouling layers from surfaces.

There is a wide range of literature that addresses the role of tangential shear stress on the removal of small particles from surfaces. Hubbe (1985) carried out flow cell experiments and demonstrated that smaller particles need higher wall shear stress for removal. The work of Sharma et al. (1992) and Yiantsios and Karabelas (1995) on the mechanism of particle removal suggest that wall shear stress induces rolling of particles.

It has been observed from experiments that pulsatile flow can be used to improve the cleaning of whey protein layers in dairy systems and fouling layers in heat exchangers. Gillham et al. studied the effect of flow pulses introduced on steady laminar flow on cleaning. They concluded that pulsing has the effect of raising the maximum shear stress at the wall thus increasing the cleaning rate. While a higher shear rate can be obtained by increasing flow rate, Gillham et al. showed that the same effect can be achieved at lower velocities by using pulsations. Bode et al. evaluated the role of use of intermittent pulsing superimposed on a steady flow to enhance the rate of cleaning of whey protein layer. The authors define a waviness parameter, which is the ratio of superimposed oscillatory velocity to the average mean velocity. They showed by experiments that as the waviness increases beyond one, cleaning time reduces.

Recent research on pulsed flow address the question under what process conditions a disproportionately higher wall stress can result when compared to a steady flow. For example, Celnik et al. solved the specific case of steady pressure gradient with superimposed periodic triangular profile as earlier experiments were performed by Gillham et al. for these conditions. For this condition, they calculated the enhancement in the wall shear stress vis-à-vis the increase in the flow rate and showed that there can be an enhancement of up to ca. 7 times in the wall shear stress with an increase in the mean flow rate of only 2.25 times. In a subsequent paper (Qi et al., 2008), the authors propose that the

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Green function solution methodology that they have developed can be applied to optimize parameters in the cleaning of microfluidic devices.

While there is enough literature evidence for the use of pulsatile flow for cleaning of whey protein and fouling layers through guiding calculations and experiments, governing framework to evaluate enhancement in wall shear stress vis-à-vis the power input have not been provided. Our objective is to provide a framework to evaluate pulsatile flow with respect to steady state in the context of particle removal. To do this, we consider a model problem of laminar flow in a plane channel with an oscillatory pressure input. We have ignored any flow instabilities arising due to pulsation and assume that the flow maintains its laminar behaviour. We carry out a systematic analysis of this problem and evaluate conditions under which pulsatile flow can be superior, i.e., more energy efficient, for particle removal from surfaces. In a practical application, say, cleaning of pipes, one would employ steady flow superimposed with an oscillatory component. The results from this analysis, which is based on purely oscillatory pressure input with no net flow, can therefore be applied for situations wherein the oscillatory component of the flow dominates over the steady component.

The organization of this paper is as follows: We consider the governing equations for oscillatory flow in a plane channel and carry out an analysis for the ratio of average wall shear stress for oscillatory vs. steady state at constant power input. Though maximum shear stress plays a key role in removal we have considered the average wall shear stress as this provides a conservative condition for removal over a period of time. We draw limiting predictions at low and high frequency. Also, from the detailed analysis, we obtain conditions for oscillatory frequency beyond which particle removal is expected to be energy efficient. We also extend this analysis to pipe flow and obtain similar results.

2. Physics of the problem

The problem under consideration is one dimensional flow in a plane channel with sinusoidal imposed pressure gradient. Fig. 1 shows a schematic representation of the flow which is in the 'x' direction with variation in the transverse 'y' direction. Under the assumption that the flow is incompressible, Newtonian and fully developed, the governing equation of motion simplifies to

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho^*}\frac{\partial p^*}{\partial x^*} + \nu^*\frac{\partial^2 u^*}{\partial y^{*2}} \tag{2.1}$$

where the superscript '*' refers to dimensional variable. The boundary condition is $u^*(y^* = \pm H^*) = 0$. Here 't^*' is the dimensional time, 'ν^*' is the kinematic viscosity, 'ρ^*' is the density of the fluid and 'p^*' is the pressure. An imposed pressure gradient that is oscillatory can be written as

$$-\frac{1}{\rho^*}\frac{\partial p^*_{osc}}{\partial x^*} = K^*_{osc}\sin\omega^* t^* \tag{2.2}$$

where K^*_{osc} is the ratio of the amplitude of pressure gradient to the density of fluid. The governing equations can be non-dimensionalized

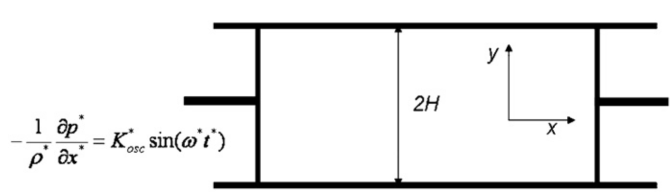

Fig. 1. Schematic representation of oscillatory flow.

as follows:

$$y = y^*/H^*, \quad t = t^*/(1/\omega^*), \quad u = u^*/(U_0^*),$$
$$K_{osc} = K^*_{osc}/(\nu^*\omega^*/H^*), \quad p_{osc} = p^*_{osc}/(\mu^*\omega^*)$$

Here U_0^* is the velocity scale which is $H^*\omega^*$ for oscillatory flow. This results in the following scaled equations:

$$Wo^2\frac{\partial u_{osc}}{\partial t} = -\frac{\partial p_{osc}}{\partial x} + \frac{\partial^2 u_{osc}}{\partial y^2} \tag{2.3}$$

$$-\frac{\partial p_{osc}}{\partial x} = K_{osc}\sin t \tag{2.4}$$

with $u(y = \pm 1) = 0$. Here 'Wo' stands for Womersley number, $Wo^2 = (H^{*2}\omega^*/\nu^*)$ is the ratio of oscillatory to viscous forces. The subscript 'osc' refers to oscillatory pressure input conditions. As we will be interested in wall shear stress, τ, and energy input/unit time/unit area (power dissipated per unit surface area), E, it would be useful to introduce scaling for these variables too

$$\tau = \frac{\tau^*}{(\mu^* U_0^*/H^*)}, \quad E = \frac{E^*}{(\mu^* U_0^{*2}/H^*)}$$

The solution for Eq. (2.3) with imposed pressure gradient Eq. (2.4) is well known (Schlichting and Gersten, 1999) and is expressed in scaled equations as follows:

$$u_{osc}(y,t) = -\frac{K_{osc}}{Wo^2}\,\mathrm{Re}\left[e^{i\,t}\left\{1 - \frac{\cosh[yWo\sqrt{i}]}{\cosh[Wo\sqrt{i}]}\right\}\right] \tag{2.5}$$

where 'Re' stands for real part of the complex number and 'i' is $\sqrt{-1}$.

Fig. 2 shows a typical plot of dimensionless velocity profile as a function of Womersley number, Wo. For fixed fluid properties and channel width, varying Wo is equivalent to varying the frequency of oscillation. In this figure, to obtain velocities that are comparable, parameter K_{osc} has also been varied arbitrarily along with Wo. An interesting observation is that as the frequency of oscillation is enhanced, the location of maximum velocity shifts from the centre of the pipe towards the wall. The flow changes from parabolic at low frequency to almost "Bingham-fluid like" at high frequency, i.e., the velocity gradient is limited close to the wall with a very flat profile at the centre. In other words, the viscous boundary layer shrinks with an increase in frequency and this scales as $\sqrt{\nu^*/\omega^*}$. This result has interesting implications for particle removal from surfaces. For the removal of particles from

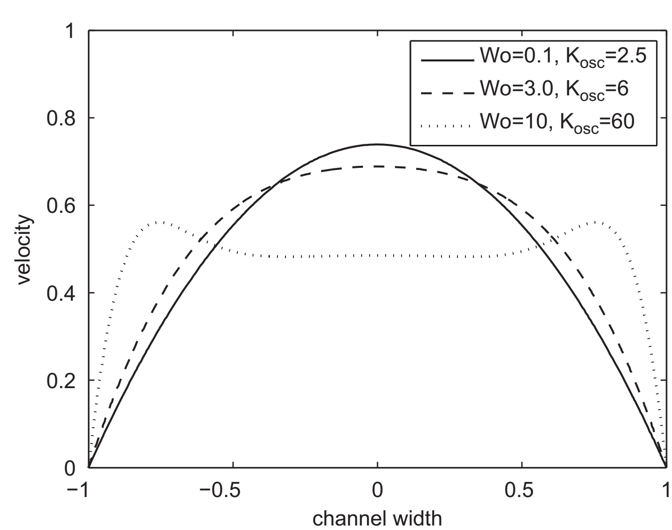

Fig. 2. Velocity profile as a function of Womersley number at time $t = 0.8\pi$.