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Integrated production planning and scheduling using a decomposition framework

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ABSTRACT

To ensure the consistency between planning and scheduling decisions, the integrated planning and scheduling problem should be addressed. Following the natural hierarchy of decision making, integrated planning and scheduling problem can be formulated as bilevel optimization problem with a single planning problem (upper level) and multiple scheduling subproblems (lower level). Equivalence between the proposed bilevel model and a single level formulation is proved considering the special structure of the problem. However, the resulting model is still computationally intractable because of the integrality restrictions and large size of the model. Thus a decomposition based solution algorithm is proposed in this paper. In the proposed method, the production feasibility requirement is modeled through penalty terms on the objective function of the scheduling subproblems, which is further proportional to the amount of unreachable production targets. To address the nonconvexity of the production cost function of the scheduling subproblems, a convex polyhedral underestimation of the production cost function is developed to improve the solution accuracy. The proposed decomposition framework is illustrated through examples which prove the effectiveness of the method.

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1. Introduction

Production planning and scheduling belong to different decision making levels in process operations, they are also closely related since the result of planning problem is the production target of scheduling problem. The traditional strategy for solving planning and scheduling problems is to follow a hierarchical approach in which the planning problem is solved first to define the production targets. The scheduling problem is solved next to meet these targets and there is no interaction between the two decision levels. In such a traditional strategy, the planning model is typically a linear and simplified representation, which is used to predict production targets and material flow over several months (up to 1 year). Scheduling models on the other hand tend to be more detailed in nature, assuming that key decisions (production targets) have been made. This traditional strategy has several disadvantages. First, the planning decisions generated might cause infeasible schedule subproblems. Since at the planning level, effects of changeovers and daily inventories are neglected, which tends to produce optimistic estimates that can not be realized at the scheduling level, i.e., a solution determined at the planning level does not necessarily lead to feasible schedules. Second, the optimality of the planning solution cannot be ensured because the planning level problem might not provide an accurate estimation of the production cost, which should be calculated from detail tasks determined by the scheduling problem.

Therefore, there is a need to develop methodologies that can effectively integrate production planning and scheduling. The objective of integrated planning and scheduling model is to obtain *feasible* and *optimal* planning decisions (production targets) for detail scheduling operation.

However, since production planning and scheduling are dealing with different time scale, it is not easy to integrate them effectively. A major challenge towards the integration is dealing with the problem size of the resulted optimization model, where the complexity is mainly due to the scheduling problem, which is generally complex mixed integer linear programming (MILP) problem. The simplest way for addressing the integrated planning and scheduling problems is to formulate a single simultaneous planning and scheduling model that spans the entire planning horizon of interest (direct full space model). However, the limitation of this approach is that, when typical planning horizons are considered, the size of this detailed model becomes intractable, because of the potential exponential increase in the computation. To overcome the above difficulty, most of the work appeared in the literature aim at decreasing the problem scale through different types of problem reduction method and developing efficient solution strategies (Grossmann et al., 2002; Maravelias and Sung, 2008). The different approaches are summarized as follows.

The first type of methods is based on hierarchical decomposition. Through hierarchical decomposition of the integrated planning and scheduling problem, detailed scheduling constraints are not

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incorporated into the upper level aggregate planning model but information is passed from the aggregate planning problem to the detailed scheduling problems which are then solved separately. Thus, the problems that need to be solved include a relative simple planning problem and a series of scheduling subproblems. To ensure the feasibility and optimality of the solution, it is further necessary to develop effective algorithms to improve the solution using additional cuts in the planning level within an iterative solution framework (Birewar and Grossmann, 1990; Papageorgiou and Pantelides, 1996; Bassett et al., 1996; Munawar and Gudi, 2005; Erdirik-Dogan and Grossmann, 2006). Another type of methods follows the rolling horizon approach. In this type of approach, detailed scheduling models are used for a few early periods and aggregate models are used for later periods thus reducing the problem size and complexity. The production targets for the early periods are directly implemented, while the production targets for the later periods are updated along with the rolling horizon (Dimitriadis et al., 1997; Sand et al., 2000; Wu and Ierapetritou, 2007; Verderame and Floudas, 2008). Furthermore, in the campaign mode of operation, periodic scheduling strategy is developed to make the operation decisions easier and profitable. This results in big savings in operations due to effective management of frequent changes and the fact that it is easy to be implemented. In a periodic scheduling framework, the planning and scheduling integration problem is replaced by establishing an operation schedule and making it executed repeatedly (Shah et al., 1993; Schilling and Pantelides, 1999; Zhu and Majozi, 2001; Castro et al., 2003; Wu and Ierapetritou, 2004).

Instead of using the detailed scheduling model in the integrated planning, surrogate type of models can also be used to represent scheduling feasibility and production cost within an aggregated planning problem. This avoids the disadvantage of large scale and complex model which directly incorporate the detailed scheduling model into aggregating planning model. Considering the fact that the lower level scheduling problems follow the same formulation, and only the production targets are different, parametric programming technique can be used to derive surrogate model of the scheduling problem which is represented by a set of critical regions and corresponding optimal production cost functions and feasibility boundary (the union of all the critical regions compose the feasible set of the scheduling problem). In this direction, Ryu et al. (2004) proposed a bilevel programming model in order to capture the dynamics of supply chain activities and further extend its application to enterprise-wide supply chain planning in (Ryu, 2005). In another type of surrogate method, Sung and Maravelias (2007) proposed a model that uses off-line computations based on a detailed scheduling model to generate the convex hull of feasible region of feasible production targets and a convex underestimation of total production cost. For this type of methods, the common difficulty is that both the feasible space and optimal objective function are non-convex functions of the planning decision parameters (production targets) and the computational efforts for an accurate description of the surrogate model depends on the dimension of the decision space (i.e., the number of products).

As briefly outlined in the previous paragraphs, a major issue in the existing methods for the solution of the integrated planning and scheduling problem lies on the fact that when the problem dimension increases, the computation complexity increases greatly. In this paper, we propose a general decomposition based solution framework to solve the integrated production planning and scheduling problem based on a bilevel optimization formulation of the integrated problem. To ensure the scheduling feasibility requirement, the production target is modeled through a penalty model, where a penalty term is incorporated into the objective function of the scheduling model to penalize the unsatisfied production targets. To develop surrogate model for the scheduling problem and also improve the accuracy of the approximation of the product cost function, the decomposition framework utilizes a type of tightest convex polyhedral underestimation of the production cost of the scheduling problem, which improves the approximation of the true cost function in an iterative framework to improve the optimality of the planning decision solution.

The rest of this paper is organized as follows. The integrated planning and scheduling problem is first described and a bilevel optimization model is presented with a structural properties analysis of the mathematical formulation in Section 2. Based on these results, a decomposition based solution framework is presented in Section 3, whereas the tightest convex polyhedral underestimation of the cost function is proposed in Section 4. Finally, two example problems and results are presented in Section 5, followed by the summary of the paper in Section 6.

2. Problem statement

Following the natural hierarchy of planning and scheduling operations, the integrated planning and scheduling model can be formulated as a bilevel optimization problem, where the upper level problem represents the planning problem, which involve the total cost minimization and is constrained by general balance equations; whereas multiple lower level problems correspond to scheduling subproblems in different planning periods, which are modeled by production cost minimization and detail scheduling constraints.

2.1. Planning model

The upper planning level problem can be formulated as following:

$$\min \quad TotalCost = \sum_t InventoryCost^t + BackorderCost^t + ProductionCost^t \tag{1}$$

$$\text{s.t.} \quad I_s^t = I_s^{t-1} + P_s^t - D_s^t, \quad \forall s \in S_P,\ \forall t \tag{2}$$

$$U_s^t = U_s^{t-1} + Dem_s^t - D_s^t, \quad \forall s \in S_P,\ \forall t \tag{3}$$

$$P_s^t, I_s^t, D_s^t, U_s^t \geq 0, \quad \forall s \in S_P,\ \forall t \tag{4}$$

$$InventoryCost_t = \sum_s h_s I_s^t, \quad \forall t \tag{5}$$

$$BackorderCost_t = \sum_s u_s U_s^t, \quad \forall t \tag{6}$$

In the planning level problem, the objective function is the total cost which is composed by three parts: inventory cost, backorder cost and production cost, where the inventory cost and backorder cost are calculated through Eqs. (5) and (6) based on the inventory and backorder amount and the given unit cost parameter (h_s, u_s); the production cost of different planning periods is determined through the lower level scheduling subproblems. Eq. (2) represents the inventory balance and Eq. (3) represents the backorder balance.

2.2. Scheduling model

In the lower level scheduling formulation, the continuous time formulation used for batch process scheduling proposed by Ierapetritou and Floudas (1998) is used as follows:

$$\min \quad ProductionCost^t = \sum_i \sum_j \sum_n (FixCost_i w_{ijn}^t + VarCost_i b_{ijn}^t) \tag{7}$$

$$\text{s.t.} \quad \sum_{i \in I_j} wv_{i,j,n}^t \leq 1, \forall j \in J, \quad \forall n \in N,\ \forall t \tag{8}$$