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Measurement of velocity and density profiles in discharging conical hoppers by NMR imaging

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ABSTRACT

Nuclear magnetic resonance (NMR) imaging was used to measure velocity and density profiles in 3-D conical hoppers fed from an open vertical silo. Discharge of a 200 μm-diameter powder in both mass and plug flow was studied with hoppers of different half angles, of 10° and 80°, respectively. An analytical solution for compressible (variable density) mass flow in the 3-D axi-symmetric geometry was also developed following the procedure outlined in Tardos (1997) and Tardos and Mort (2005). The density variation and velocity profiles obtained experimentally were compared to predictions of this theory for dense, compressible granular flows. We found, from both theory and experiment, that the powder has to exhibit significant dilation (compressibility) as it is accelerated through the constriction in the hopper. The degree of compressibility was found, experimentally, to be lower than that predicted by the mass flow hopper theory. The powder unexpectedly exhibited a boundary layer with a fully-rough boundary condition in the mass flow hopper. In the funnel-flow hopper, the expected "dead zone" was found around the orifice and extended about one diameter length into the silo. The centerline velocity increased according to an exponential function.

1. Introduction

Despite major advances in the theory of powder statics and dynamics, the rate of freely discharging powders from hoppers cannot be computed theoretically with a precision better then a factor of between two and four. For the case of very fine and cohesive powders such effects as the influence of the interstitial gas and adverse pressure gradients across the outlet make the problem much more complicated and some deviation form theoretical calculations is acceptable. However, for cohesion-less materials of relatively large size (well above 100 μm in diameter) and mostly spherical shape where the problem should be straightforward, such large deviations from theory are unacceptable.

The rate of discharge from a 3-D axi-symmetric hopper was measured experimentally by several researchers (see Elbicki and Tardos, 1998, and Tardos and Mort, 2005 for an extensive review) and the accepted equation to predict this flow rate, was established by Beverloo, (in Shamlou (1990)) and reads:

$$W = C\rho_{bulk}\sqrt{gD_h^5}, \quad \text{with } D_h = D_o - 1.4d_p \tag{1}$$

Here W, is the mass flow-rate, ρ_{bulk}, is the material's bulk density, g is the acceleration of gravity and D_{h}, is a nominal diameter of the outlet calculated from the geometric diameter D_0 and the particle size, d_{p}, as shown in Eq. (1). The equation contains a coefficient, C, taken by Beverloo as, $C = 0.58$. The value of the discharge rate appears to be insensitive to the hopper half angle, material's coefficient of internal and wall friction and in general to any physical property of the powder (aside from its density) as long as cohesion and interstitial gas do not come into play.

Several researchers have attempted to solve analytically and numerically the outflow from the hopper (see extensive review in Tardos (1997)). One of the simpler solutions, the so called hour-glass theory (see Nedderman, 1992) reads:

$$W = \frac{\pi}{4}\sqrt{\frac{1}{(5\sin\phi - 1)\sin\alpha}}\rho_{bulk}\sqrt{gD_h^5} \tag{2}$$

where in addition to the notations in Eq. (1), ϕ, is the angle of internal friction and α, is the hopper half angle as shown in Fig. 1. Thus, the coefficient is computed theoretically as $C=(\pi/4)[(5\sin\phi-1)\sin\alpha]^{-1/2}$; unfortunately, for a typical value of the internal powder friction angle, such as 30°, this expression gives elevated values; $C > 1$ for hopper angles below 25°. One should note that the theoretical approach leading to Eq. (2) assumes the powder to be incompressible, i.e., ρ_{bulk} is assumed to be constant. Tardos (1997) also obtained

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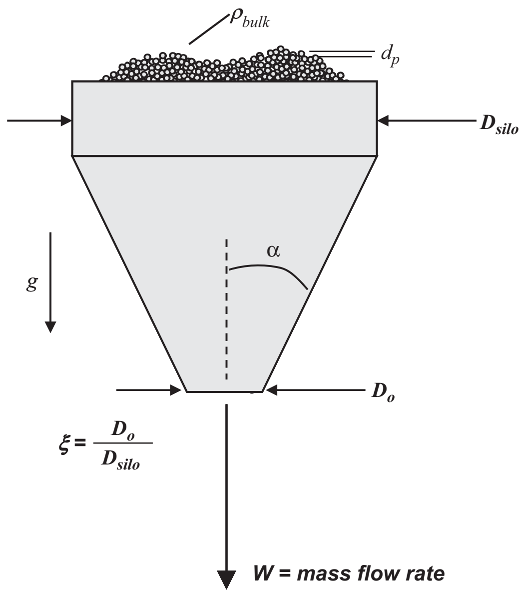

Fig. 1. Schematic representation of the axi-symmetric hopper/silo geometry.

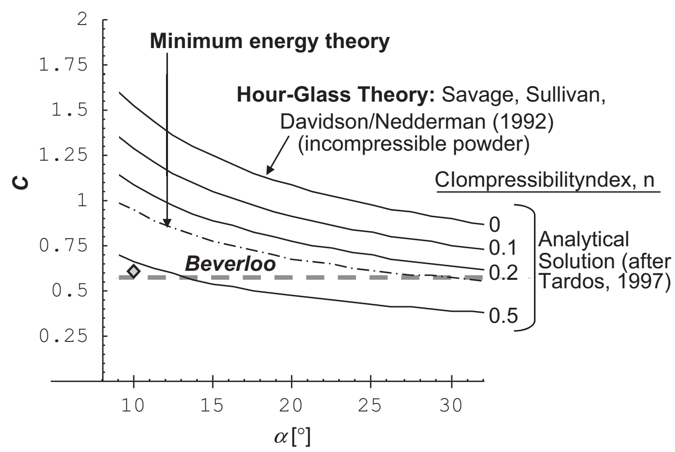

Fig. 2. Experimental result and calculated mass flow-rates (coefficients C) as a function of the hopper half angle, α, from several theoretical models. The prediction of the Beverloo equation is also shown. The compressible curves ($n = 0.1$–0.5) are computed with $\phi = 30^\circ$ and $\xi = 0 \cdot 19 \cdot$ The hour-glass solution has $\phi = 30^\circ$.

a similar solution from the continuum-based equations proposed by Schaeffer (1987) for an incompressible powder obeying the von Mises yield criteria.

A recent numerical solution by Shrivastava and Sundaresan (2003) who took into account the existence of velocity fluctuations in the powder, by introducing the effect of the granular temperature and some form of powder compressibility, calculated for the coefficient C the value $C = 1.6$. While there are still a large number of assumptions in both models cited above, including somewhat unrealistic boundary conditions, the discrepancy between theory and measurements is too large to be simply overlooked.

There is little experimental data available for fully 3-D (conical) hoppers. Tuzun et al. have reported low-resolution density profiles in mass flow hoppers with 10° and 30° half angles (Hosseini-Ashrafi and Tuzun, 1993; Langston et al., 1997, respectively). Observed dilation was under predicted by discrete element simulation in the latter work.

We attempt in this note to (i) measure powder flow rates from conical hoppers fed from a cylindrical bin with small, 200 μm-diameter, fairly round particles, (ii) measure axial velocity and density fields in the conical sections using NMR and (iii) compare the flow rate and the velocity and density profiles to a theoretical model obtained from the equations developed by Schaeffer (1987) but taking into account powder compressibility.

2. Theoretical analysis

We follow closely the method of solution described in detail in Tardos (1997) for the *compressible powder* and the geometry of the 2-D hopper and apply it to the 3-D axi-symmetric case. We reproduce here the set of equations that need to be solved for this geometry. We start with the continuity equation that reads:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{u}) = 0 \quad (3)$$

where ρ, denotes the variable bulk density of the material. This equation is satisfied approximately by an appropriate choice of the velocity and density variation in the hopper, in the form:

$$\bar{u} = v_r(r) = \frac{A}{r^{2+n}}; \quad \text{and } \rho = \rho_{bulk}\left(\frac{r}{R}\right)^n \quad (4)$$

Here, r is the radial coordinate measured from the virtual apex of the hopper-cone and R is the length of the cone to the point where the cylindrical silo is connected to the conical hopper so that the maximum value of $r_{\max} = R$, as shown in Fig. 5. For moderate to large friction angles, the choice of density distribution as shown in Eq. (4), is equivalent to a powder compressibility law of the form (Tardos, 1997):

$$p(\rho)^{-1/n} = const. \quad (5)$$

holding over much of the contraction where p depends linearly on radius. It is worthwhile noting that $n = 1$ for an ideal (granular) gas but n should be small for a solid state for which large pressures are necessary for small changes in the powder volume.

The constant A in Eq. (4) has to be determined from the momentum conservation equations given below:

$$\rho \frac{D\bar{u}}{Dt} = \nabla p - \nabla \cdot \left(\sqrt{2} q(p,\rho) \frac{D_{ij} - \nabla \cdot \bar{u}\delta_{ij}/3}{|D_{ij} - \nabla \cdot \bar{u}\delta_{ij}/3|}\right) + \rho \bar{g} \quad (6)$$

The notations in this equation are those used in fluid mechanics to denote D/Dt, as the material derivative, δ_{ij} the unit tensor (the Kronecker delta) and p as the pressure (average normal stress in this case). The only special feature of this equation is the yield condition given by the function $q(p,\rho)$ which in this case is taken as $q \approx p \sin \phi$, the well-known Coulomb friction yield condition. Assuming, in addition, that the pressure is only a function of the radius, $p = p(r)$, and that the hopper's half angle is relatively small (steep hopper) so that the action of gravity is $\vec{g} = g_r$, yields an analytical solution for the velocity in Eq. (4) with the constant A, given as:

$$A = f(n, \sin \phi)\xi^{2+n} \sqrt{\frac{\xi - \xi^{\frac{6\sqrt{3}\ \sin \phi}{3-2\sqrt{3}\ \sin \phi} - n}}{1 - \xi^{1+n+\frac{9}{3-2\sqrt{3}\ \sin \phi}}}} \quad (7)$$

where the quantity, ξ, is the normalized centerline apex-to-outlet distance, expressible as D_o/D_{silo}, and "f" is a known function of the index "n" and the internal angle of friction of the powder. The above solution assumes open hopper boundary conditions ($p = 0$ at $r/R = \xi$ and 1), as depicted in Fig. 1. A silo solution, with upper boundary condition $p'(r) = 0$, gives a slightly more cumbersome expression that agrees quantitatively, within a few percent, for modest $\xi(\leq 0 \cdot 2)$ and realistic friction angles—as expected.