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Size effect in fatigue based on the extreme value distribution of defects

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ABSTRACT

Fatigue limits need to be extrapolated from test specimens to manufactured products. The relevant industry standards provide a method for this by utilizing the statistics of defects in the material. We show here that the standard method involves an inappropriate definition. Moreover, it relates to the characteristic size of the largest defects, which is not associated with any unique exceedance probability. We outline a more consistent method which, by a quantile of the largest defects, relates the sample size effect to the desired failure probability. This method is applicable also to samples smaller than the test specimen.

1. Introduction

Fracture and fatigue tests are normally done for small test specimens. The data need to be applied to estimating fatigue in manufactured components of various dimensions. Thus, determining the size effect, i.e. extrapolating from test specimens to objects with other dimensions is critically important in the engineering design [1,2].

Depending on the application, material, and loading conditions, different fatigue mechanisms may dominate and sometimes also compete with each other. Factors, such as local microstructure, local stress/strain, 3D-shape, macro/micro-topography, grain orientation and proximate defects, determine if a defect will form a crack. An in-depth modeling shows that cracks do not always initiate at the deepest flaw or largest defect [3,4]. The challenge will be to use the governing factors and damage physics to understand the process [5,6]. Thus, a purely statistical approach to the size effect is not an accurate representation of the fatigue failure mechanisms.

However, it is recognized that the industry standards and the extreme value approach must use simplifying assumptions to provide useful engineering approximations. This paper originates from studies concerning high strength steels subjected to high cycle fatigue, where crack initiation from subsurface inclusions dominates [7]. This mechanism is relevant for many rotating or reciprocating components, e.g. in automotive industry. As such, the statistical approach of this paper is generic, i.e., not limited to a particular failure mechanism or field of industry.

When the fatigue limit is strongly correlated with the defect size distribution in the material, the extrapolation from small specimens can be done by statistics of the largest defects and applying the extreme value theory [2,8–14]. In this connection the defect size distribution is considered to be a property of the material and thus independent of the object size. Furthermore, it is considered that the amount of defects in all samples is sufficiently large so that the theory of extremes may be applied.

A detailed method of determining the sample size effect by extreme statistics of defects is given in the international standard ASTM Designation E 2283-03 [12] and the discussion document of ESIS-TC20 [13]. In this paper, we critically discuss this method and propose an alternative procedure that we find more appropriate and applicable to practical fatigue design.

2. The standard method

The method of determining the effect of sample size, given in the ASTM and ESIS documents, utilizes the concept of return period T. This concept originates from geophysical sciences where the annual extremes of natural variables are of interest. The value X_T which the annual maximum of the variable x exceeds once in the mean in T years is called the return value. Correspondingly, the period T is called the return period of X_T.

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Nomenclature

A	surface area of the sample of interest
A_0	surface area of the control test specimen
F	cumulative distribution function
F^{-1}	quantile function (inverse of F)
m	order when observations are ranked from the smallest to the largest
n	number of observations, ratio of the larger specimen area to the smaller specimen area
P	probability of non-exceedance
P_e	probability of the annual maximum of x to exceed X_T
T	return period
x	variable value
X_P	P-quantile value of the largest defects in the sample specimen
$X_{P,0}$	P-quantile value of the largest defects in the control test specimen
X_T	return value
λ	location parameter
δ	scale parameter
σ	failure stress

In the ASTM [12] and ESIS [13] documents the return period is defined as

$$T = A/A_0 \tag{1}$$

where A is the sample area of interest and is A_0 is the area of the control test specimen set at 150 mm^2 in [12]. The method considers the sample size effect in terms of the parameter called the *characteristic largest defect size* in a surface area of the sample. This is the defect size which has an expectation to be exceeded once in that area.

The return period T is, by its definition, equal to $T = 1/P_e$ where P_e is the probability of the annual maximum of x to exceed X_T. Hence,

$$T = 1/(1-P) \tag{2}$$

where $P = 1-P_e$ is the corresponding non-exceedance probability. Based on the theory of unlimited extremes, the non-exceedance probability of the size x of the largest defect in a sample is in the standards estimated by the Gumbel distribution [15] as

$$P(x) = \exp[-\exp(-(x-\lambda)/\delta)] \tag{3}$$

The parameters λ and δ are called the location and scale parameter, respectively. The characteristic largest defect size is, therefore, calculated in this method from Eqs. (2) and (3) as

$$x(T) = \lambda - \delta[\ln(-\ln(1-1/T))] \tag{4}$$

Physically, the definition of the defect size may vary with the type of defect. For inclusions in steel, the square root of the defect area at a cross-section of a specimen is normally used [7].

3. Problems with the standard method

We identify several problems regarding the standard approach. First, one would expect that an appropriate method defines a characteristic largest defect size at $T = 1$, i.e., when the sample surface area equals the control surface area, $A = A_0$. However, Eq. (4) has no solution at $T = 1$. When one considers a T that is close to unity, for example $T = 1.1$, then Eq. (4) yields $x(1.1) = \lambda - 0.87\delta$. This suggests, for very small size corrections, a smaller characteristic largest defect size for a surface with a larger area, contrary to what one would expect qualitatively.

Furthermore, Eq. (4) has no solution when $T < 1$, i.e. when $A < A_0$. This prevents using the method in extrapolating the fatigue limit to objects or stress concentration areas that are smaller than the test specimen. While such extrapolation may not be necessary for conservative design of small components, it allows optimal cost-effective manufacturing.

Second, the concept of the return period T of a value X_T originates from geophysical sciences, where T is defined as the time in years during which an annual maximum exceeds X_T once in the mean. Geophysical measurements continue irrespectively of events of exceedance. For example, an annual wind speed may exceed a critical limit in time T many times or not at all. This is not in analogy with a failure surface of a material, where the specimen fails when a defect exceeds the critical size limit. Thus, at a failure surface the critical defect size is exceeded exactly once. It thus appears that the concept of return period and the related Eq. (4) are inappropriate when considering detailed defect statistics of failure surfaces.

Third, the probability of not exceeding X_T within a period of length T can be calculated from the probability theory and is

$$P(X_T) = (1-1/T)^T \tag{5}$$

For $T = 3$, as an example, Eq. (5) gives $P = 0.30$. By the analogy used in the standard method, the characteristic largest defect size is then exceeded in an area $A = 3A_0$ with the probability of $P_e = 0.70$. For, say, $A = 10A_0$ the corresponding probability is $P_e = 0.65$. Thus, the probability of including, in a sample, a defect larger than the characteristic largest defect size of that sample depends on the size of the sample. This dependence, shown in Fig. 1, is not strong for large size corrections. Nevertheless, it makes it difficult to apply this method to optimizing industrial products so that their design failure probability is predetermined and independent of the specimen size.

Further still, in fatigue life estimates by surface areas the return period should, instead of Eq. (1), be defined as [16–18]

$$T = (A + A_0)/A_0 \tag{6}$$

Eq. (1), used in the standards [12,13] and in [7], is inappropriate for this reason too.

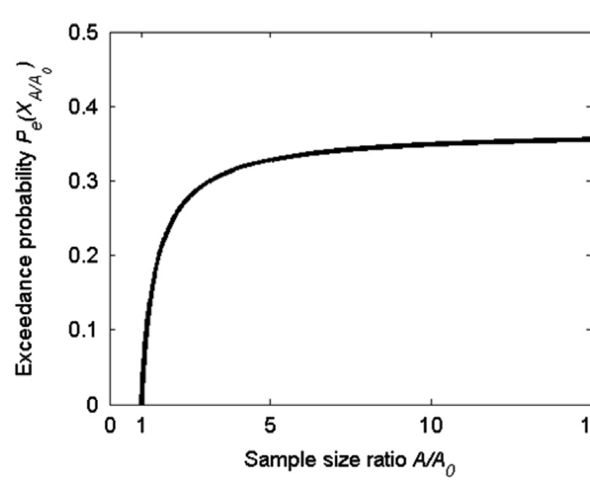

Fig. 1. Probability that the characteristic largest defect size of an area is exceeded in that area based on the ASTM standard method [12], Eqs. (1) and (5).