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Continuous-time Wiener-model predictive control of a pH process based on a PWL approximation

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ABSTRACT

This paper deals with a novel formulation of continuous-time model-predictive control for nonlinear systems. A nonlinear-mapping approximation, employing a PWL approximation, is also an integral part of the control scheme, and thus removes the need for output-function invertibility. The analytical formulation of the control law makes it possible to use the method in practice, especially in the chemical industry. An illustrative experiment is conducted to compare the proposed approach with the method of nonlinear H_∞ control of a pH-neutralization process.

1. Introduction

Linear-model predictive control (LMPC) is a well-known and well-established industry standard. The generic term LMPC refers to a class of control algorithms in which a linear dynamic process model is used to predict and optimize process performance. Having its roots in the 1970s, LMPC is nowadays commercially available primarily for controlling constrained multivariable processes (Henson, 1998). However, many processes are sufficiently nonlinear to render the successful application of LMPC impossible. There are two main cases where a nonlinear type of control has to be considered: a moderately nonlinear process with large operating regimes (e.g., multi-grade polymer reactors) and a highly nonlinear process that operates near a fixed operating point (high-purity distillation columns). The need for high-quality control of such processes led to the development of the nonlinear-model predictive control (NMPC) methods. In general, NMPC (and also LMPC) is an optimization-based control strategy where a sequence of control moves is computed to minimize an objective function that includes predicted future values of the controlled outputs. The predictions in each computation step are obtained from a nonlinear process model. The majority of methods rely on the so-called nonlinear receding horizon principle (Mayne and Michalska, 1990; Henson, 1998), where feedback is included by implementing only the manipulated inputs computed for the present time step, then moving the prediction horizon forward one step and repeating the procedure with the new measurements. This strategy yields an open-loop optimal controller. There is a wide variety of existing NMPC methods, for example in Maner et al. (1996), Badgwell (1997) and Norquay et al. (1999) in the discrete-time framework, and in Demircioglu and Gawthrop (1991), Chen et al. (2003) and Magni and Scattolini (2004) in the continuous-time framework. For the state of the art of NMPC methods the reader is referred to the papers by Morari and Lee (1999) and Henson (1998). Issues like the stability and optimality of the NMPC methods were discussed in detail by Mayne et al. (2000).

NMPC requires the availability of a suitable nonlinear dynamic model of the process, and the accuracy of the model is of paramount importance. There are two general classes of nonlinear models used: fundamental models, based on transient mass, energy, and momentum balances, and empirical models, such as Hammerstein, Wiener, Volterra, and fuzzy models, which are derived on the basis of empirical data from the process. The majority of NMPC methods are derived in discrete time, and therefore need discrete-time models. On the other hand, the majority of models are given in continuous form and need to be discretized. The drawbacks of discretizing nonlinear continuous-time models were discussed by Pearson (2003); and they include structural changes, the dependence of the stability on the model's parameters and the initial states, and the inaccurate system intersample behavior (Magni and Scattolini, 2004). Because of this our proposed method is based on a continuous-time model of a

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process, and the model can be obtained either directly from a first-principle nonlinear process model or by identification.

The proposed approach in this paper tackles the control of nonlinear processes using continuous-time nonlinear model-based predictive control. The advantage of the proposed approach is in the continuous-time framework which is used here. This means that the sampling time, when applying the algorithm on digital hardware and the digital redesign is made, can be selected after the analogue control system is designed and, thus the continuous-time closed-loop bandwidth is known. The approach in continuous-time enables also a multi-sampling-rate digital implementation.

Special attention is given to the processes that can be efficiently modelled using a Wiener-type model. The linear dynamics are derived from the first-principle nonlinear model; and the nonlinear output function is calculated from the steady-state equations and approximated by piecewise-linear (PWL) functions. The model-output prediction is calculated using a Taylor-series expansion of the Wiener model. The Wiener-PWL structure makes it possible to form the prediction in an exact analytical form consisting of a constant linear, and a variable nonlinear, part. The former is a constant matrix that can be calculated off-line, and the latter is a scalar product representing the gradient of the PWL-approximation of the nonlinearity in the Wiener model. The receding-horizon strategy was combined with a cost function that minimizes the difference between the future-output-prediction error and the model-prediction error. Consequently, this brings the following benefits to the control-law calculation:

- The law is derived in a closed analytical form, which resolves the issue of nonlinear optimization and achieving the global optimum in each calculation step.
- The reduction of the calculation of the control signal to a scalar product and an inverse of a scalar (everything else can be calculated offline) brings a significant reduction in the computational complexity. This makes it possible to consider the proposed method for practical applications.
- The nonlinearity is inherently included in the law; compared to the method in Norquay et al. (1999); this removes the need to invert the NL approximation.

The outline of the paper is as follows. In Section 2 the PWL functions are introduced and the model-output prediction is formulated in the continuous-time domain. In Section 3 the nonlinear predictive control law is derived, and some stability issues are also discussed. Section 4 presents the pH-neutralization process and gives a comparison of the closed-loop-control results for the proposed approach and a nonlinear H_∞ approach. Section 5 concludes the paper with some directions for future work.

2. Problem statement

Let us assume a nonlinear continuous-time system

$$\dot{x}_p(t) = f(x_p(t), u(t)),$$

$$y_p(t) = g(x_p(t)),$$

$$y_p(0) = y_{p0}, \quad y_p(T) = y_{pT}, \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}$ are smooth functions, $x_p \in \mathbb{R}^n$ is a vector of n state variables, $u \in \mathbb{R}$ is a process input and $y_p \in \mathbb{R}$ is a process output. An optimal control can in general be seen as the solution of

$$\min_{u \in \Omega} J(x_p, y_p, t) = \min_{u \in \Omega} \int_0^T F(x_p, y_p, t)\, dt, \tag{2}$$

where $F \in \mathbb{R}^+$ is a cost function that satisfies the optimality criteria, and Ω is the set of admissible control signals. In other words, we have to design a controller that asymptotically stabilizes a closed-loop system in such a way that the process output, $y_p(t)$, optimally follows the prescribed reference trajectory, $y_r(t)$, according to the given performance index J. However, dealing with nonlinear continuous-time systems, the problem setup in (2) leads to solving the Hamilton–Jacobi–Bellman partial differential equations (Bertsekas, 1995). The solution of the HJB PDE system is usually obtained numerically (Chen et al., 2003), which is computationally too expensive to be considered for practical control applications. As an alternative, in this paper we avoid solving the system of PDE by using the moving-horizon control concept (Mayne and Michalska, 1990; Clarke et al., 1987; Chen et al., 2003). The idea is to calculate the optimal control sequence in each time instant by minimizing the given performance index, which involves open-loop prediction of the model output and the predicted reference signal. The initial conditions are the reference, the model output and the process measured output at the given time instant t, and the closed-form analytical solution is open-loop optimal. After applying the calculated input signal $u(t)$, the time-frame is moved to the next time instant.

2.1. Dealing with a nonlinearity in a system by using the Wiener model and a PWL approximation

The system's nonlinearity presents an additional difficulty in terms of system modelling and control. This problem can be successfully solved by using a Wiener-type system that has a special structure that facilitates its application to model-based predictive control. The Wiener system has the structure of a dynamic linear block followed by a static nonlinearity

$$\dot{x}(t) = Ax(t) + Bu(t),$$

$$v(t) = Cx(t),$$

$$y(t) = h(v(t)), \tag{3}$$

where $A \in \mathbb{R}^n \times \mathbb{R}^n$, $B \in \mathbb{R}^n$, and $C \in \mathbb{R}^n$ are the state-space matrices, $h : \mathbb{R} \to \mathbb{R}$ denotes the static nonlinear mapping and $y \in \mathbb{R}$ is the process-model output. The variable $v(t) \in \mathbb{R}$ represents the intermediate variable that does not necessarily have a clear physical meaning. Notice also that the functions h from (3) and g from (1) are not necessarily equal because, in general, the static nonlinearity in the model also covers the effects of the nonlinearity in the states of the process. Different approaches to Wiener-model identification are found in the literature. The most frequently used is the nonlinear–linear (N–L) approach, which is the most comprehensible and ensures an accurate description of the static nonlinearity (Gerkšič et al., 2000). This approach requires steady-state data. The excitation signal has to be designed to obtain the information about the steady-state behavior of the system. The steady-state curve of the observed system is obtained from data pairs of the input variable u and the corresponding output variable y during steady-state, (u_{si}, y_{si}). The data set of steady-state points is a non-equidistant set of data and it is spread around the nominal static curve. This set of steady-state points is now modelled using PWL approach.

Using the PWL approximation, the process-model output is defined as

$$y(t) = \hat{h}(v(t)) = \Theta^T \Lambda(v(t)), \tag{4}$$