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Estimation of tensile load-bearing capacity of ductile metallic materials weakened by a V-notch: The equivalent material concept

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ABSTRACT

Ductile commercial steel was equated from the viewpoint of strain energy density with a virtual brittle material of the same elastic modulus by using a novel concept, called the equivalent material concept (EMC). By determining the ultimate tensile strength of the virtual material and also, by assuming that the values of the plane–strain fracture toughness for real and virtual materials are equal, the tensile load-bearing capacity of several V-notched samples of steel reported in literature was theoretically estimated by using the mean stress (MS) criterion as a well-known brittle fracture theory. It was found that the theoretical results of the MS-EMC criterion for the imaginary brittle specimens are in a very good consistency with the experimental results reported for real steel samples.

1. Introduction

Unlike cracks which are usually unfavorable to appear in engineering components and structures, notches, particularly V-shaped ones, are utilized because of special design requirements. A V-notch concentrates stresses around its tip and hence can become prone to initiate cracks. Such cracks can propagate in the notched component and finally lead to fracture. From the viewpoint of fracture mechanics, the mechanisms of crack initiation and propagation from a notch tip are fundamentally different for ductile and brittle materials. For brittle materials, the emanation of crack from the notch tip consumes a great portion of the total fracture energy and the crack propagation is very less contributed in energy consumption. This is because the crack growth is such a rapid and unstable phenomenon that the final fracture occurs suddenly. Conversely, for ductile materials exhibit large plastic deformations around notches, both crack initiation and propagation consume considerable amount of energy during ductile rupture.

Fracture in notched metallic materials under fatigue loading conditions has been widely studied by several investigators during which the number of cycles to failure could be predicted (see for example [1–4]). However, the number of researches in open literature dealing with fracture in ductile metallic materials containing notches under static and monotonic loading conditions is very limited. For instance, J-integral has been evaluated under elastic–plastic conditions by Berto et al. [5] as a governing parameter in fracture assessment of U and V-notched components made of ductile materials obeying a power-hardening law. Susmel and Taylor [6] have published a paper dealing with predicting the load-bearing capacity for a type of ductile steel containing notches of different features by using the theory of critical distances (TCD) under pure tensile loading conditions (i.e. pure mode I deformation). Their tested material has been a commercial cold-rolled carbon steel exhibiting very ductile behavior. The notched specimens tested by them showed large plastic deformations around the notches after fracture [6]. They predicted the maximum load that each notched specimen can sustain by performing linear elastic and elastic–plastic stress analyzes in conjunction with the use of the theory of critical distances (TCD) with a maximum discrepancy of about 15%. They have clearly stated in their paper [6] that the good efficiency of TCD in the presence of large plastic deformations around notches is surprising. Also, stated in [6] is that the justification of the satisfactory accuracy of TCD accompanied by linear elastic stress analysis is very complicated for ductile V-notched samples. Although the experimental results reported in [6] have been in a good agreement with the results of TCD, its application in engineering design together with linear elastic analysis cannot be prescribed from the viewpoint of fracture mechanics principles. A set of elastic–plastic analyzes have also been performed in [6] accompanied by TCD to predict the load-bearing capacity of the

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Nomenclature

$K_I^{V,\rho}$	notch stress intensity factor (NSIF)-mode I
K_{Ic}^V	mode I notch fracture toughness
2α	notch angle
λ_1	eigenvalue
μ_1	eigenvalue (real parameter)
σ_{rr}	radial stress
$\sigma_{r\theta}$	in-plane shear stress
$\sigma_{\theta\theta}$	tangential stress

notched specimens under pure mode I loading conditions which have finally led to good consistency with experimental results. Since elastic–plastic analyzes are rather time-consuming and relatively complicated with respect to elastic analyzes, the author preferred to suggest a simple failure criterion to be conveniently used in the failure analysis of ductile notched domains.

Two of the first attempts to use simple elastic analysis instead of complicated elastic–plastic analysis have been made by Glinka and Molski [7] and Glinka [8]. They made use of the strain energy density (SED) approach for determining the elastic–plastic stress distribution around several notched components. In their works, first, the elastic stress concentration factor has been utilized to formulate the elastic stresses at the notch tip and then, the SED at the notch tip has been equated for elastic and elastic–plastic components in order to determine the stress distribution in the notched component made of ductile material. The equivalent strain energy density approach has been reformulated and applied to sharp V-shaped notches under localized and generalized plasticity by Lazzarin and Zambardi [9] for predicting the failure of components containing V-shaped notches. They used the strain energy over a finite volume around the notch tip as a governing failure parameter to predict the onset of failure in several notched specimens [9].

In this research, the mean stress (MS) criterion, proposed and formulated by Ayatollahi and Torabi [10] for predicting the onset of mode I brittle fracture in V-notched components, is employed in conjunction with the novel equivalent material concept (EMC) (presented in forthcoming sections) to estimate the experimental results reported in [6] for the tensile load-bearing capacity of several V-notched specimens made of ductile steel. It was found that the theoretical predictions of the MS-EMC criterion for the V-notched samples imaginarily fabricated from the virtual brittle material (i.e. the equivalent material) agreed well with the experimental results reported for real steel samples.

2. The equivalent material concept and the mean stress fracture criterion

2.1. The equivalent material concept

In this section, a novel concept, called the equivalent material concept (EMC), is introduced and utilized with the aim to equate a ductile material with a virtual brittle material from the viewpoint of strain energy density. By using EMC, one can imaginarily consider in fracture investigations a virtual brittle material exhibiting linear elastic behavior instead of the ductile material with elastic–plastic behavior. Finally, brittle fracture criteria may be simply utilized to study the fracture phenomenon in ductile materials.

The approach of EMC suggested and utilized herein in order to predict the load-bearing capacity of V-notched components made of ductile materials is relatively similar to that suggested by Glinka [8] but with fundamentally different target. According to the EMC, the strain energy density (i.e. the area under the stress–strain curve

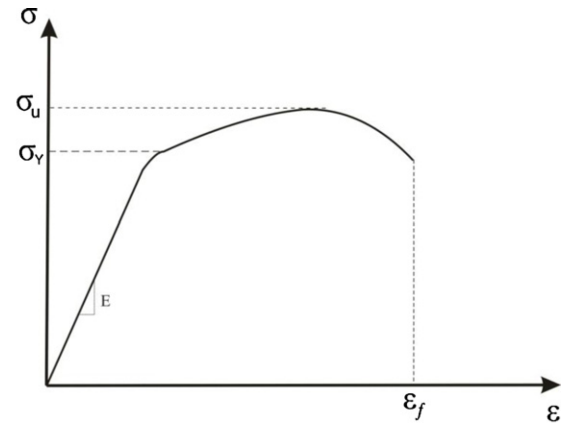

Fig. 1. A sample tensile stress–strain curve for a typical ductile material.

in uni-axial tension) for the existing ductile material is assumed to be equal to that for a virtual brittle material having the same modulus of elasticity. The strain energy density (SED) is, in fact, the strain energy absorbed by a unit volume of material. For a ductile material with considerable plastic deformations and with exhibiting power-law strain-hardening relationship in the plastic zone, one can write

$$\sigma_P = K\varepsilon_P^n \quad (1)$$

In Eq. (1), σ_p and ε_p are the plastic stress and the plastic strain, respectively. The parameters K and n are also the strain-hardening coefficient and exponent, respectively which depend upon the material properties. Fig. 1 displays schematically a tensile stress–strain curve for a typical ductile material.

In Fig. 1, E, σ_Y, σ_u and ε_f denote the elastic modulus, the yield strength, the ultimate tensile strength and the strain to rupture, respectively. The total SED can be written in a general form of elastic–plasticity as

$$(\mathrm{SED})_{\mathrm{tot.}} = (\mathrm{SED})_e + (\mathrm{SED})_p = \frac{1}{2}\sigma_Y\varepsilon_Y + \int_{\varepsilon_Y}^{\varepsilon_p} \sigma_p d\varepsilon_p \quad (2)$$

Substituting $\varepsilon_Y = \sigma_Y/E$ and Eq. (1) into Eq. (2) gives

$$(\mathrm{SED})_{\mathrm{tot.}} = \frac{\sigma_Y^2}{2E} + \int_{\varepsilon_Y}^{\varepsilon_p} K\varepsilon_p^n d\varepsilon_p \quad (3)$$

Thus

$$(\mathrm{SED})_{\mathrm{tot.}} = \frac{\sigma_Y^2}{2E} + \frac{K}{n+1}(\varepsilon_P^{n+1} - \varepsilon_Y^{n+1}) \quad (4)$$

If ε_Y is considered to be equal to 0.002 (corresponding to 0.2% offset yield strength), then

$$(\mathrm{SED})_{\mathrm{tot.}} = \frac{\sigma_Y^2}{2E} + \frac{K}{n+1}(\varepsilon_P^{n+1} - (0.002)^{n+1}) \quad (5)$$

In order to calculate the total SED corresponding to the onset of crack initiation (i.e. the area under σ–ε curve from beginning of loading to σ_u), one can replace ε_p in Eq. (5) with ε_u (i.e. the strain at maximum load)

$$(\mathrm{SED})_{\mathrm{tot.}} = \frac{\sigma_Y^2}{2E} + \frac{K}{n+1}(\varepsilon_u^{n+1} - (0.002)^{n+1}) \quad (6)$$

From Eq. (1), ε_u can be obtained as

$$\varepsilon_u = \left(\frac{\sigma_u}{K}\right)^{1/n} \quad (7)$$