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Extensive data of notch shape factors for V-notched Brazilian disc specimen under mixed mode loading

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ABSTRACT

The linear elastic stress field was numerically analyzed around the V-notch tip in a recently developed disc-type test specimen, called the V-notched Brazilian disc (V-BD), under mixed mode in-plane loading. The notch stress intensity factors (NSIFs) which are very important parameters in brittle fracture assessment of V-notched components were calculated for V-BD specimen by using the finite element (FE) method for different notch geometries and wide range of mode mixities from pure mode I to pure mode II loading conditions. In order to simplify the obtained results to be used in practical applications, the NSIFs were converted to the dimensionless parameters called the notch shape factors (NSFs). These parameters are useful to compute more rapidly and conveniently the NSIFs in V-BD specimen for various notch angles and different notch tip radii. It is shown that the notch shape factors presented in this work combined with the appropriate failure criteria can be utilized to estimate the load-cell capacity of the test apparatus required for fracture test of V-BD specimens made of different brittle materials.

1. Introduction

Different failure modes can be recognized in brittle and quasi-brittle materials like ceramics, rocks, brittle polymers, concrete, graphite, soda-lime glass etc. among which the brittle fracture mode is very popular. In the presence of stress concentrators like cracks and notches, brittle materials are so vulnerable to failure. Therefore, many researchers have frequently investigated the brittle fracture in engineering structures containing stress concentrations both theoretically and experimentally.

Various types of notches, especially V-shaped ones, are usually utilized in engineering components and structures because of particular design requirements. V-notches act as stress raisers and reduce significantly the load-bearing capacity of the notched component due to stress concentration around the notch tip. If the notched component is made of a brittle material and subjected to mechanical loading, fracture may occur suddenly from the notch. Therefore, it is essential to prevent brittle fracture in engineering applications by using appropriate failure criteria and/or by conducting fracture tests.

Similar to cracks, V-shaped notches can be generally subjected to three different in-plane loadings usually called pure mode I, pure mode II and mixed mode I/II. Under pure mode I loading, notch faces open with respect to the notch bisector line without any sliding. Any pure in-plane sliding of the notch faces with respect to the notch bisector line is called pure mode II deformation and combined opening and in-plane sliding of the notch faces is well-known as mixed mode I/II deformation.

Several failure theories can be found in literature for predicting brittle fracture in notched elements that most of them have been developed based on the linear elastic fracture mechanics (LEFM) such as that presented by Sih and Ho [1] based on the critical energy density theory and those suggested on the basis of the local strain energy density concept (see for example [2–7]). Like cracks, the stress intensity factors play an important role in governing the brittle fracture phenomenon in notched components. For instance, the notch stress intensity factors (NSIFs) have been utilized in [8–10,15,16] to predict the onset of brittle fracture in V-notched and U-notched components, respectively.

Experimental investigations dealing with determining the fracture toughness (i.e. the critical values of the NSIFs) of notched elements have been performed by several researchers using a limited number of test samples. For example, one can find in literature the single-edge notch tension (SENT) [17], double-edge notch tension (DENT) [18], three-point bend (TPB) [19] and four-point bend [20] specimens. Recently, Ayatollahi and Torabi [12] suggested and used a new disc-type test specimen containing a rhombic central slit and tested under compressive loading, called the V-notched Brazilian disc (V-BD) specimen, for measuring the notch fracture toughness of plexi-glass (PMMA) [12], polycrystalline graphite [13]

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Nomenclature

$K_{\mathrm{I}}^{V,\rho}$	notch stress intensity factor (NSIF)-mode I
$K_{\mathrm{II}}^{V,\rho}$	notch stress intensity factor (NSIF)-mode II
K_{Ic}^{V}	mode I notch fracture toughness
RNL	relative notch length
RNR	relative notch tip radius
V-BD	V-notched Brazilian disc
$Y_{\mathrm{I}}^{V,\rho}$	notch shape factor (NSF)-mode I
$Y_{\mathrm{II}}^{V,\rho}$	notch shape factor (NSF)-mode II
2α	notch angle
β	loading angle for V-BD specimen
β_{II}	loading angle corresponding to pure mode II loading
λ_i	eigenvalues
μ_i	eigenvalues (real parameters)
σ_{rr}	radial stress
$\sigma_{r\theta}$	in-plane shear stress
$\sigma_{\theta\theta}$	tangential stress

and soda-lime glass [14] under various combinations of mode I and mode II loadings. The V-BD specimen is, in fact, a modified version of the centrally cracked Brazilian disc (CCBD) specimen which has been frequently utilized in mixed mode I/II fracture tests of brittle components containing sharp cracks (see for example [21–23]).

In this work, 276 finite element (FE) analyzes were performed to calculate the notch stress intensity factors (NSIFs) for V-BD specimen under different mixed mode loading conditions from pure mode I to pure mode II for different notch angles and various notch tip radii. For more simplicity of using the obtained results in practical applications, the NSIFs are converted to the dimensionless parameters, called the notch shape factors (NSF). The NSFs which depend on the specimen diameter, specimen thickness, notch length, notch angle and the notch tip radius are presented in several suitable graphs. Once the NSFs and the geometric parameters are known for a V-BD specimen, one can directly determine the NSIFs for any combination of modes I and II loadings without requiring FE analysis. In the next section, closed-form expressions are presented in a general form for the linear elastic stress distribution around the tip of a rounded-tip V-notch. These expressions are then utilized to define and calculate the NSIFs and the NSFs in the forthcoming sections.

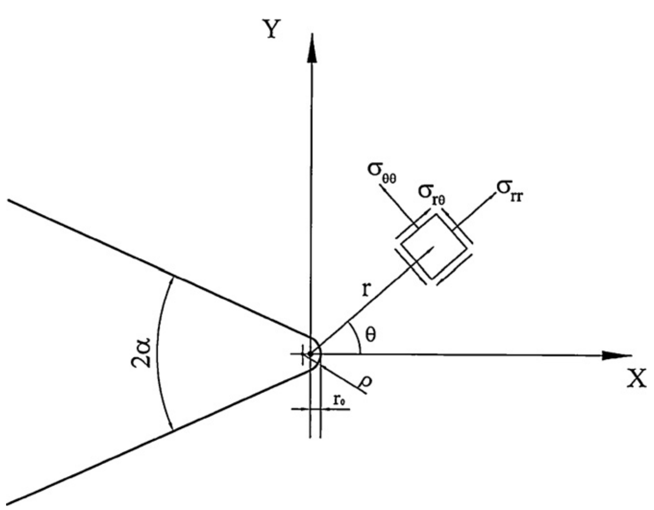

Fig. 1. Round-tip V-notch and its polar coordinate system.

2. Linear elastic stress distribution around a V-notch tip

Filippi et al. [24] developed an expression for mixed mode I/II stress distribution around a V-shaped notch shown in Fig. 1. The stress distribution is an approximate formula because it satisfies the boundary conditions only in a finite number of points on the notch edge and not on the entire edge. They obtained the stress distribution using a conformal mapping in an auxiliary system of curvilinear coordinates "U and V" that are related to the Cartesian coordinates "X and Y" as $(X+iY)=(U+iV)^q$ [24]. The power "q" is a real positive coefficient ranging from 1 (for a flat edge) to 2 (for a crack).

The mixed mode I/II stresses can be written as

$$\begin{Bmatrix}\sigma_{\theta\theta}\\ \sigma_{rr}\\ \sigma_{r\theta}\end{Bmatrix} = \frac{K_{\mathrm{I}}^{V,\rho}}{\sqrt{2\pi}r^{1-\lambda_1}}\left[\begin{Bmatrix}f_{\theta\theta}(\theta)\\ f_{rr}(\theta)\\ f_{r\theta}(\theta)\end{Bmatrix}^{(\mathrm{I})} + \left(\frac{r}{r_0}\right)^{\mu_1-\lambda_1}\begin{Bmatrix}g_{\theta\theta}(\theta)\\ g_{rr}(\theta)\\ g_{r\theta}(\theta)\end{Bmatrix}^{(\mathrm{I})}\right] + \frac{K_{\mathrm{II}}^{V,\rho}}{\sqrt{2\pi}r^{1-\lambda_2}}\left[\begin{Bmatrix}f_{\theta\theta}(\theta)\\ f_{rr}(\theta)\\ f_{r\theta}(\theta)\end{Bmatrix}^{(\mathrm{II})} + \left(\frac{r}{r_0}\right)^{\mu_2-\lambda_2}\begin{Bmatrix}g_{\theta\theta}(\theta)\\ g_{rr}(\theta)\\ g_{r\theta}(\theta)\end{Bmatrix}^{(\mathrm{II})}\right] \quad (1)$$

where $K_{\mathrm{I}}^{V,\rho}$ and $K_{\mathrm{II}}^{V,\rho}$ are the mode I and mode II notch stress intensity factors (NSIFs), respectively. The parameter r_0 is the distance between the origin of the polar coordinate system and the notch tip. The functions $f_{ij}(\theta)$ and $g_{ij}(\theta)$ have been reported in the Appendix and the eigenvalues λ_i and μ_i which depend upon the notch angle have been reported in [24]. It can be shown that if the notch tip radius vanishes, Eq. (1) becomes equal to the stress field previously obtained by Williams [25] for sharp V-notches. According to a relation that exists between the Cartesian and the curvilinear coordinate systems, r_0 can be written as [24]:

$$r_0 = \frac{q-1}{q}\rho, \quad q = \frac{2\pi-2\alpha}{\pi} \quad (2)$$

where "2α" is the notch angle and ρ is the notch tip radius. The expressions for NSIFs are [26]:

$$K_{\mathrm{I}}^{V,\rho} = \sqrt{2\pi}\frac{(\sigma_{\theta\theta})_{\theta=0}r^{1-\lambda_1}}{1+\omega_1(r/r_0)^{\mu_1-\lambda_1}} \quad (3)$$

$$K_{\mathrm{II}}^{V,\rho} = \sqrt{2\pi}\frac{(\sigma_{r\theta})_{\theta=0}r^{1-\lambda_2}}{1+\omega_2(r/r_0)^{\mu_2-\lambda_2}} \quad (4)$$

where $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$ are the tangential and the in-plane shear stresses, respectively. The parameters ω_1 and ω_2 have been presented in Appendix A. If the values of ω_1 and ω_2 are known, the NSIFs can be obtained from Eqs. (3) and (4) as

$$K_{\mathrm{I}}^{V,\rho} = \sqrt{2\pi}\frac{\sigma_{\theta\theta}(r_0,0)\,{r_0}^{1-\lambda_1}}{1+\omega_1} \quad (5)$$

$$K_{\mathrm{II}}^{V,\rho} = \lim_{r\to r_0}\sqrt{2\pi}\frac{\sigma_{r\theta}(r,0)\,r^{1-\lambda_2}}{1-(r/r_0)^{\mu_2-\lambda_2}} \quad (6)$$

The NSIFs can be calculated by using the FE method as elaborated in the next section. These parameters have different units of measure (MPa $\mathrm{m}^{(1-\lambda_1)}$ and MPa $\mathrm{m}^{(1-\lambda_2)}$ for $K_{\mathrm{I}}^{V,\rho}$ and $K_{\mathrm{II}}^{V,\rho}$, respectively) and hence, cannot directly be compared. Note that the parameter r in Eq. (6) cannot be directly substituted by r_0, because for $r=r_0$, $K_{\mathrm{II}}^{V,\rho}$ becomes singular. Therefore, $K_{\mathrm{II}}^{V,\rho}$ is calculated from Eq. (6) at a point very close to the notch tip where $r\to r_0$. When the notch tip radius ρ is zero (i.e. in the case of a sharp notch), the stress values at the notch tip tend to infinity and hence the parameters $K_{\mathrm{I}}^{V,\rho}$ and $K_{\mathrm{II}}^{V,\rho}$ cannot be directly obtained by using Eqs. (5) and (6). In such conditions, the limits of the expressions given in Eqs. (5) and (6) must be calculated where $r\to 0$. It should be highlighted that the NSIFs are very important parameters in brittle fracture assessment of V-notched components because, wide range of the