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A drag correlation of fluid particles rising through stagnant liquids in vertical pipes at intermediate Reynolds numbers

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ABSTRACT

A drag correlation for a fluid particle rising along the axis of a vertical pipe at low and intermediate Reynolds numbers, *Re*, is proposed by making use of available correlations and a numerical database accumulated by interface tracking simulations. The accuracy of the interface tracking method has been verified through comparisons between measured and predicted velocities of single drops in vertical pipes. Being similar to drag model for solid spheres proposed by Michaelides, the developed drag correlation takes into account inertial and wall effects as their linear combination. The correlation gives good estimation of the drag coefficient for fluid particles rising through stagnant liquids in vertical pipes under the conditions of $0.13 \leqslant Eo \leqslant 30$, $-10.0 \leqslant \log M \leqslant 2.0$, $0.083 \leqslant Re < 200$, $0 \leqslant \kappa \leqslant 10.0$ and $\lambda \leqslant 0.6$, where *Eo* is the Eötvös number, *M* the Morton number, κ the viscosity ratio and λ the ratio of particle diameter to pipe diameter.

1. Introduction

Reliable models for forces acting on fluid particles such as drag, lift and virtual mass forces are indispensable in accurate prediction of dispersed multiphase flows using multifluid models (Lehr and Mewes, 2001; Tomiyama and Shimada, 2001; Wiemann and Mewes, 2005) and Eulerian–Lagrangian methods (Žun et al., 1993; Delnoij et al., 1997; Laín et al., 1999; Tomiyama et al., 2003). Though a number of studies on the forces acting on fluid particles in infinite stagnant liquid have been carried out (Levich, 1962; Clift et al., 1978; Magnaudet and Eames, 2000; Tomiyama, 2004), our knowledge on the effects of pipe walls on the forces acting on fluid particles is still insufficient.

For single fluid particles rising or falling through stagnant liquids in vertical pipes, the wall effects on drag force have been taken into account (Uno and Kintner, 1956; Strom and Kintner, 1958; Haberman and Sayre, 1958; Harmathy, 1960; Wallis, 1969; Clift et al., 1978). Haberman and Sayre (1958) derived a drag model for fluid particles at low Reynolds numbers. They expressed the wall effect in terms of the diameter ratio, which is the ratio of the particle diameter to the pipe diameter, and the viscosity ratio. On the other hand, for high Reynolds number fluid particles, the increase in drag has been modeled as a function of the diameter ratio only (Uno and Kintner, 1956; Strom and Kintner, 1958; Wallis, 1969; Clift et al., 1978). However there are few drag models applicable to fluid particles in pipes at intermediate Reynolds numbers.

The purpose of this study is, therefore, to develop a drag correlation for a single fluid particle rising through a stagnant liquid in a vertical pipe at low and intermediate Reynolds numbers. Interface tracking simulations are carried out by using the non-uniform subcell scheme (NSS) (Hayashi et al., 2006; Hayashi and Tomiyama, 2007) to examine the validity of the developed drag correlation. Physical experiments of drops in pipes are also conducted to verify the accuracy of NSS.

2. Derivation of a drag correlation from available models

Let us review available drag models for solid spheres, bubbles and drops to develop a drag correlation for single fluid particles rising through stagnant liquids in vertical pipes at low and intermediate Reynolds numbers.

For spherical fluid particles rising through infinite stagnant liquid at low Reynolds numbers, Hadamard (1911) and Rybczynski (1911) derived the following drag coefficient C_{D0}:

$$C_{D0} = \frac{8(2+3\kappa)}{Re_0(1+\kappa)} \tag{1}$$

where Re_0 is the Reynolds number of a fluid particle in infinite stagnant liquid and κ the viscosity ratio. They are defined by

$$Re_0 = \frac{\rho_C V_{T0} d}{\mu_C} \tag{2}$$

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$$\kappa = \frac{\mu_D}{\mu_C} \tag{3}$$

where ρ is the density, V_{T0} the terminal velocity of a single fluid particle in infinite stagnant liquid, d the sphere-volume equivalent diameter of a fluid particle, μ the viscosity and the subscripts C and D denote the continuous and dispersed phases, respectively. Taking the limit of $\kappa \to \infty$ yields the Stokes' drag law for a solid sphere:

$$C_{D0} = \frac{24}{Re_0} \tag{4}$$

For a spherical bubble in a pure system, the viscosity ratio is close to zero ($\kappa \approx 0$) and Eq. (1) reduces to

$$C_{D0} = \frac{16}{Re_0} \tag{5}$$

Schiller and Nauman (1933) extended the Stokes' drag law by multiplying the following drag factor, K_{Is}, which accounts for the inertial effect on the drag force acting on a solid sphere in infinite stagnant liquid at intermediate Reynolds numbers:

$$C_{D0} = \frac{24}{Re_0} K_{Is} \tag{6}$$

Here K_{Is} is given by

$$K_{Is} = 1 + 0.15\, Re_0^{0.687} \tag{7}$$

Tomiyama et al. (1998) and Myint et al. (2006) confirmed that K_{Is} is appropriate not only for solid spheres but also for spheroidal bubbles and spheroidal drops rising through infinite stagnant liquid at intermediate Reynolds numbers, i.e., the drag models for spheroidal bubbles and drops are given by

$$C_{D0} = \frac{16}{Re_0} K_{Is} \tag{8}$$

$$C_{D0} = \frac{8(2 + 3\kappa + 3C/\mu_C)}{Re_0(1 + \kappa + C/\mu_C)} K_{Is} \tag{9}$$

where C is a coefficient expressing the retardation of interface motion by surfactants (Levich, 1962).

The terminal velocity, V_T, of a fluid particle rising along the axis of a vertical pipe is smaller than V_{T0} since the presence of the pipe wall increases the drag, i.e., $C_D/C_{D0} > 1$, where C_D is the drag coefficient of a fluid particle in the pipe. The relationship between C_D and C_{D0} for fluid particles in low viscosity systems has been modeled in terms of the diameter ratio λ, which is defined by

$$\lambda = \frac{d}{D} \tag{10}$$

where D is the pipe diameter. For example, Clift et al. (1978) proposed the following simple empirical correlation of the wall effect on drag:

$$C_D = C_{D0} K_{Wc} \tag{11}$$

where K_{W_c} is the wall effect multiplier defined by

$$K_{Wc} = (1 - \lambda^2)^{-3} \tag{12}$$

Eq. (11) is valid for $\lambda < 0.6$, the Eötvös number $Eo < 40$ and $Re > 200$, where Re is the Reynolds number of a fluid particle in a pipe defined by

$$Re = \frac{\rho_C V_T d}{\mu_C} \tag{13}$$

The Eötvös number Eo is defined by

$$Eo = \frac{g \Delta\rho d^2}{\sigma} \tag{14}$$

where g is the acceleration of gravity, $\Delta\rho$ the density difference ($\Delta\rho = |\rho_C - \rho_D|$) and σ the surface tension. For low-Reynolds number spherical fluid particles rising along the pipe axis, Haberman and Sayre (1958) derived the following theoretical model:

$$C_D = \frac{8(2 + 3\kappa)}{Re(1 + \kappa)} K_{Wh} \tag{15}$$

This model is a combination of the Hadamard–Rybczynski solution, Eq. (1), and the wall effect multiplier K_{Wh}, which is a function of λ and κ:

$$K_{Wh}(\lambda, \kappa) = \left(1 + c_0 \lambda^5 \frac{1-\kappa}{2+3\kappa}\right)\left(1 + c_1 \frac{2+3\kappa}{1+\kappa}\lambda + c_2 \frac{\kappa}{1+\kappa}\lambda^3 + c_3 \frac{2-3\kappa}{1+\kappa}\lambda^5 + c_4 \frac{1-\kappa}{1+\kappa}\lambda^6\right)^{-1} \tag{16}$$

where $c_0 = 2.2757$, $c_1 = -0.7017$, $c_2 = 0.20865$, $c_3 = 0.5689$ and $c_4 = -0.72603$, respectively. Eq. (15) gives good estimation of C_D for λ up to about 0.5 even though a drop deforms due to the presence of the pipe wall (Clift et al., 1978).

There are no drag models applicable to fluid particles in pipes at intermediate Reynolds numbers. For spherical solid particles, Michaelides (2006) suggested that C_D at intermediate Reynolds numbers is well expressed as the following linear superposition of the inertial and wall effects, provided that $\lambda \leqslant 0.8$ and $Re \leqslant 20$:

$$C_D = \frac{24}{Re}(K_I + K_W - 1) \tag{17}$$

where K_I and K_W are the drag factors for the inertial and wall effects, respectively. The maximum error is 15%. For higher λ and Re, the linear superposition may not be valid. Assuming that the above linear superposition also holds for fluid particles in pipes at low and intermediate Reynolds number, we obtain the following drag correlation:

$$C_D = \frac{8(2 + 3\kappa)}{Re(1 + \kappa)}[K_{Wh}(\lambda, \kappa) + 0.15\, Re^{0.687}] \tag{18}$$

in which Eqs. (1), (7) and (16) are used for the base drag coefficient, inertial effect and wall effect, respectively. Since $K_{Wh}(\lambda, \kappa)$ in Eq. (18) is applicable for $\lambda < 0.5$, Eq. (18) may be also valid for $\lambda < 0.5$. The applicable ranges of Eq. (18) will be discussed later.

Fluid particles rising along the pipe axis were simulated using the interface tracking method based on the NSS in two-dimensional cylindrical (r, z) coordinates to examine the validity of the speculated correlation, Eq. (18). Physical experiments on drops in pipes were also conducted to verify the accuracy of NSS.

3. Numerical method

A brief introduction of the interface tracking method, NSS, is given in this section. The details can be found in Hayashi et al. (2006).

3.1. Field equations

The following mass and momentum equations based on one-field formulation for an incompressible two-phase flow are adopted (Hirt and Nicholds, 1981; Scardovelli and Zaleski, 2000):

$$\nabla \cdot \boldsymbol{u} = 0 \tag{19}$$

$$\rho\left[\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u}\right] = -\nabla P + \nabla \cdot \mu[\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T] + \rho \boldsymbol{g} + \sigma \mathrm{K} \boldsymbol{n} \delta \tag{20}$$

where $\boldsymbol{u}$ is the velocity, t the time, P the pressure, $\boldsymbol{g}$ the acceleration of gravity, K the mean curvature of the interface, $\boldsymbol{n}$ the unit normal to the interface and δ the delta function. The density and viscosity