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Theory of plasticity and anelasticity due to dislocation creep through a multi-scale hierarchy of obstacles

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ABSTRACT

The creep of a dislocation on its glide plane is essentially controlled by three different stress fields: the external applied stress, the internal stress field due to a multi-scale hierarchy of different obstacles (the structural defects acting on the dislocation by short- or long-range interactions) and the thermal stress field due to thermal fluctuations. The dislocation glide dynamics involves solution of a string equation, which can be written as a Langevin equation. In this paper, it is shown that general analytical solutions of this equation can be found, allowing calculation of the plastic strain rate and the amplitude-dependent internal friction (*ADIF*), by using simple assumptions concerning the multi-scale hierarchy of obstacles and the mechanisms of Brownian dislocation creep through the different kinds of interacting obstacles. It is also shown that several experimental observations are well explained by this approach.

1. Introduction

Several experimental observations related to dislocation creep are not well explained, for instance: (i) the strong similarity of the temperature dependence in a lot of different materials between the yield stress σ_{yield} obtained by tensile experiments and the stress amplitude σ_0 allowing a constant product $\sigma_0 \cdot IF$ obtained by amplitude-dependent internal friction (*IF*) measurements [1], (ii) the very different behavior of the internal friction background observed in different measurement frequency ranges, (iii) the behavior of the r value (the ratio between the internal friction and the modulus defect) observed during amplitude-dependent internal friction (*ADIF*) measurements [2], and (iv) the "strange" temperature dependence of the internal friction background in certain materials ($IF \propto \exp(\eta T)$) [3,4].

In the following, it will be shown that these experimental observations are well explained by some general considerations and solutions of the Langevin equation describing the Brownian dislocation creep through a multi-scale hierarchy of obstacles.

2. Brownian dislocation creep

2.1. Langevin equation of dislocation creep

In order to describe the creep of a dislocation on its glide plane (Fig. 1) when it is submitted to the internal stress field due to obstacles (structural defects interacting with it), an equation taking into account the thermal fluctuations can be obtained by using an analogy with the *Langevin equation* [5] describing the *Brownian motion* of an over-damped particle in a viscous fluid:

$$m\dot{v} + \zeta v = F_{appl} + \tilde{F}(t) \quad (1)$$

in which m is the inertial mass of the particle; ζv is the stochastic resistance due to the fluid viscosity, related to the thermal fluctuations via the fluctuation–dissipation theorem; F_{appl} is the external force applied to the particle and $\tilde{F}(t)$ is the random thermal fluctuating force exerted by the surroundings.

As dislocations are linear objects which are also over-damped by a stochastic resistance due to phonon interaction (this has been experimentally verified in metals by the existence of a phonon-dislocation damping relaxation in the MHz range [6–8]), a *Langevin string model* of the dislocation motion [9] can be written for the displacement $u(x, t)$ of the dislocation line on its glide plane, by

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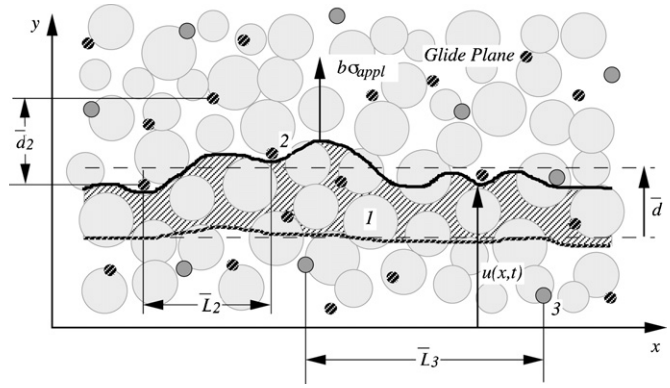

Fig. 1. Schematic representation of the *macro-creep* $u(x, t)$ of a dislocation on its glide plane, through an internal stress field $\sigma_{int}(x, y)$ generated by a three scale-levels hierarchy of obstacles, under the effect of an applied stress σ_{appl}.

analogy with Eq. (1):

$$\left(M_d \frac{\partial^2 u}{\partial t^2} + B_{ph} \frac{\partial u}{\partial t}\right) \sqrt{1 + \left(\frac{\partial u}{\partial x}\right)^2} - \frac{\partial}{\partial x}\left[\frac{\gamma(\partial u/\partial x)}{\sqrt{1 + (\partial u/\partial x)^2}}\right]$$
$$= b\sigma_{appl} + b\sigma_{\mathrm{int}}(x, u(x, t)) + b\tilde{\sigma}_{th}(x, u(x, t), t) \quad (2)$$

in which $\gamma = \alpha\mu b^2$ is the *line tension* of the dislocation, which is due to the elastic deformation energy stored in the crystal by the dislocation, with b the *Bürgers vector* of the dislocation, μ the *shear modulus* and α a constant; $M_d = \gamma c_t^2$ is the mass *per unit length of dislocation* line, which is due to relativistic effects, with c_t the shear wave velocity; $B_{ph} = B_{ph}(T)$ is the *damping or drag coefficient due to phonons and electrons*, which depends on temperature T [13]; $b\sigma_{appl}$ is the *Peach–Koehler force* per unit length of dislocation line, due to the external applied stress σ_{appl}; $\sigma_{int}(x, y)$ is the *internal stress field* along the dislocation line ($y = u(x, t)$) due to the different obstacles (the interacting structural defects) and $\tilde{\sigma}_{th}(x, y, t)$ represents the *thermal stress field fluctuations* along the dislocation line ($y = u(x, t)$) due to the acoustical and/or the optical phonons.

Solving directly such a complicated Eq. (2) without numerical simulations is quite impossible. But it will be shown in the following that one can find general analytical solutions of it, allowing one to calculate the plastic strain rate and the amplitude-dependent internal friction, by using some simple assumptions concerning the multi-scale hierarchy of obstacles and the mechanisms of Brownian dislocation creep through each of the different kinds of interacting obstacles.

2.2. Multi-scale hierarchy of interacting obstacles

When a dislocation is submitted to an external applied stress σ_{appl}, its motion on the glide plane is controlled by the internal stress field $\sigma_{int}(x, y)$ generated by the obstacles (all the different interacting structural defects). This stress field can present a regular distribution on the glide plane (for instance in the case of the Peierls potential due to the crystallographic structure of the crystal [10]) or random spatial fluctuations (for instance in the case of an interaction with point obstacles randomly distributed on the glide plane or structural defects distributed in the bulk [11]).

Considering all the different structural defects that can interact with the dislocation leads to the existence of a *multi-scale hierarchy of obstacles* on the glide plane. This obstacle hierarchy can be characterized by the total number N of different kinds of obstacles appearing at different scales in the internal stress field $\sigma_{int}(x, y)$. One can then attribute a number k to each kind of obstacles, choosing number 1 for the densest obstacles and number N for the more dispersed obstacles.

An example of such a multi-scale hierarchy for $N = 3$ is illustrated in Fig. 1: a dense distribution of weak and extended obstacles for $k = 1$ (largest clear circles), for instance due to long-range interactions with distant structural defects as point defects, a less dense distribution of stronger but less extended obstacles for $k = 2$ (smaller shaded circles), for instance due to short-range interaction with point obstacles situated near the glide plane, and very dispersed stronger defects for $k = 3$ (smaller gray circles), for instance due to some precipitates, or to the dislocation forest, or to other stronger obstacles.

2.3. Plastic strain rate due to long-range dislocation creep

During a plastic deformation experiment, if a constant stress $\sigma_{appl} = \sigma_0$ is applied, the dislocations move through the obstacles represented by the internal stress $\sigma_{int}(x, y)$ by alternating successive waiting times in front of the obstacles with *Brownian jumps* of these obstacles, arising when sudden high enough thermal fluctuations $\tilde{\sigma}_{th}(x, u(x, t), t)$ take place. This macro-creep mechanism is responsible for a *plastic strain rate* $\dot{\varepsilon}_{pl}(\sigma_0, T)$, which has to depend strongly on applied stress σ_0 and temperature T, and which is deduced from the *average plastic creep velocity* $\dot{\bar{u}}_{pl}$ of dislocations by the *Orowan relation*, in which Λ is the *density of mobile dislocations*:

$$\dot{\varepsilon}_{pl}(\sigma_0, T) = \Lambda b \dot{\bar{u}}_{pl}(\sigma_0, T) \quad (3)$$

The total time τ, which is needed, for a dislocation to move on an average distance $\bar{d}$ (as illustrated in Fig. 1) is the sum of the waiting times passed in front of all the obstacles if one neglects the short times used for the jumps. τ is also the sum of the N partial waiting times τ_k passed by the dislocation in front of all the obstacles of kind k, so that:

$$\dot{\bar{u}}_{pl}(\sigma_0, T) = \frac{\bar{d}}{\tau} = \frac{\bar{d}}{\sum_{k=1}^{N} \tau_k} = \frac{1}{\sum_{k=1}^{N} \tau_k/\bar{d}} = \frac{1}{\sum_{k=1}^{N} 1/(\dot{\bar{u}}_k(\sigma_0, T))} \quad (4)$$

This important relation shows that the average creep velocity $\dot{\bar{u}}_{pl}(\sigma_0, T)$ of a dislocation interacting with a multi-scale hierarchy of obstacles can be obtained directly from the *virtual macro-creep velocities* $\dot{\bar{u}}_k(\sigma_0, T)$ of the same dislocation interacting only with the obstacles of kind k, which allows one to write the *plastic strain rate* associated with the long-range creep as

$$\dot{\varepsilon}_{pl}(\sigma_0, T) = \frac{\Lambda b}{\sum_{k=1}^{N} 1/(\dot{\bar{u}}_k(\sigma_0, T))} \quad (5)$$

in which the *virtual macro-creep velocities* $\dot{\bar{u}}_k(\sigma_0, T)$ have to be obtained from dynamical models of long-range dislocation motion impeded only by the distribution of the obstacles of kind k, and submitted to a constant applied stress σ_0 (see examples of such models in Section 4). This relation implies in fact that the obstacles responsible for the smallest velocity $\dot{\bar{u}}_k(\sigma_0, T)$ essentially control the plastic strain rate.

2.4. Anelasticity due to “fractal” short-range dislocation creep

During internal friction measurement, a cyclic stress $\sigma_{appl}(t)$ is applied with amplitude σ_0 and frequency ω. This stress is responsible for a *short-range cyclic creep* $\bar{u}_{an}(t)$ of the dislocation segments and a cyclic anelastic strain $\varepsilon_{an}(t) = \Lambda b \bar{u}_{an}(t)$ (Fig. 2). Due to the multi-scale hierarchy of obstacles, the area swept by a dislocation segment will present some kind of *“fractal aspect”*, as illustrated in Fig. 2, so that the total anelastic strain $\varepsilon_{an}(t)$ has to be composed of several contributions:

$$\varepsilon_{an}(t) = \Lambda b \bar{u}_{an}(t) = \Lambda b \left(\bar{u}_{pl}(t) + \sum_{k=2}^{N} \bar{u}_{an}^{(k)}(t)\right) \quad (6)$$