Materials Science and Engineering A 524 (2009) 186–192

Contents lists available at ScienceDirect

Materials Science and Engineering A

journal homepage: www.elsevier.com/locate/msea

Effective strain rate sensitivity of two phase materials

László S. Tóth [a,*], Alain Molinari [a], Olivier Bouaziz [b]

[a] Laboratoire de Physique et Mécanique des Matériaux, Université de Metz, Ile du Saulcy, 57045 Metz, Moselle, France
[b] Arcelor-Mittal Research, France

ARTICLE INFO

Article history:
Received 1 April 2009
Received in revised form 10 June 2009
Accepted 19 June 2009

Keywords:
Strain rate sensitivity
Viscosity
Composites
Modeling

ABSTRACT

Analytical expressions are derived for the effective strain rate sensitivity exponent of a two phase material when the behavior of both phases and of the composite itself can be described by power law relations between the stress and strain rates. The material is assumed to be plastically isotropic and obey to von-Mises type creep behavior. Two types of boundary conditions are considered: strain or stress-controlled. The obtained formulas are applied to a geological composite material (mixture of camphor and octachloropropane) with the help of different simple models of two phase composites.

1. Introduction

Thanks to technical progress, new materials emerge with improved performance. They are used in more and more extreme conditions. Two phase materials are good examples. When they are subjected to high temperatures, their components display enhanced sensitivity to the applied strain rate. It is important to be able to predict the strain rate sensitivity of a two phase material in such conditions. The subject of this paper is to give some new formulas for the calculation of the effective strain rate sensitivity of multiphase materials.

In this work, isotropy is assumed for both phases as well as for the composite with von-Mises type viscoplastic behavior. It is well established that strain rate dependence of mechanical strength of materials can be well described by a power law relationship between the equivalent strain rate and the equivalent stress [1].

$$\bar{\sigma} = k\left(\frac{\dot{\bar{\varepsilon}}}{\dot{\varepsilon}_0}\right)^m. \tag{1}$$

Here $\bar{\sigma}$ is the Mises equivalent stress and $\dot{\bar{\varepsilon}}$ is the von-Mises equivalent strain rate corresponding to the strain rate tensor $\dot{\underline{\underline{\varepsilon}}}$:

$$\bar{\sigma} = \sqrt{\tfrac{3}{2}S_{ij}S_{ij}}, \tag{2}$$

$$\dot{\bar{\varepsilon}} = \sqrt{\tfrac{2}{3}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}. \tag{3}$$

k represents the strength of the material in the constitutive law expressed by Eq. (1) and $\dot{\varepsilon}_0$ is a reference constant (this is the strain rate at which the stress level is: $\bar{\sigma} = k$). Finally, m characterizes the strain rate sensitivity. Only the plastic part of the strain rate is used in Eqs. (1) and (3) and $\underline{\underline{S}}$ means the deviatoric stress in Eq. (2). Relation (1) is valid at a given strain (the parameter k can express strain hardening). While k normally decreases with an increase of temperature, m usually *increases*. In this way, the effect of the strain rate sensitivity becomes more and more important in the stress level that the material can sustain in a given application. However, a composite is made of two (or more) components that normally do not have the same m values in their constitutive law:

$$\bar{\sigma}_1 = k_1\left(\frac{\dot{\bar{\varepsilon}}_1}{\dot{\varepsilon}_0}\right)^{m_1}, \tag{4}$$

$$\bar{\sigma}_2 = k_2\left(\frac{\dot{\bar{\varepsilon}}_2}{\dot{\varepsilon}_0}\right)^{m_2}. \tag{5}$$

An important question arises then: what is the effective m value of the composite? Little efforts have been done so far to answer this problem. To estimate the m value, many authors simply employ the so-called rule of mixtures:

$$m = f_1 m_1 + f_2 m_2. \tag{6}$$

Here f_1 and f_2 are the volume fractions of the two phases, they sum up to 1:

$$f_1 + f_2 = 1. \tag{7}$$

* Corresponding author. Tel.: +33 387547238; fax: +33 387315366.
E-mail addresses: toth@univ-met.fr (L.S. Tóth), molinari@univ-met.fr (A. Molinari), olivier.bouaziz@arcelormittal.com (O. Bouaziz).

0921-5093/$ – see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.msea.2009.06.041

Those authors who do not wish to introduce further assumptions in their work (like relation (6)) may use an implicit numerical way to find out the effective m value (see, for example, in [2]).

In the present paper, *analytical* expressions are derived for the effective strain rate sensitivity exponent of a two phase material. Two types of boundary conditions are considered: i. strain, ii. stress-controlled conditions of testing. It will be shown that relation (6) corresponds to non-realistic boundary conditions. The new theoretical formulas are applied to a geological composite material; the mixture of camphor and octachloropropane, in the whole volume fraction range. For the mechanical behavior of this two phase material, four simple models are employed; i: the classical uniform deformation (Taylor) and the Static model, ii: the so-called Iso-W which assumes that the plastic power is uniform within the whole material [3,4] and another new model in which the macroscopic plastic power dissipation is equal to the volume fraction average of the phases (plastic-power-criterion model, 'PPC'). While the macroscopic strain rate sensitivity values obtained using Static and the two plastic power based approaches are all in good agreement with experimental results, the stress–strain rate behavior depends significantly on the model.

2. Basic hypotheses

A composite consisting of two phases is considered. Phase no. 1 is made up from inclusions while phase no. 2 is the matrix. The behavior of the phases can be described by the usual viscous power laws given by Eqs. (4) and (5) while that of the composite is assumed to be well approached by the power law (1), where m possibly depends on the strain rate $\bar{\dot{\varepsilon}}$. For simplicity, it is assumed in this work that the average stress and strain rate quantities in the phases are proportional to the macroscopic ones. Thus, the strain and stress localizations are defined by:

$$\underline{\underline{\dot{\varepsilon}}}_1 = r_1 \underline{\underline{\dot{\varepsilon}}}, \quad \underline{\underline{\dot{\varepsilon}}}_2 = r_2 \underline{\underline{\dot{\varepsilon}}}, \tag{8}$$

$$\underline{\underline{S}}_1 = t_1 \underline{\underline{S}}, \quad \underline{\underline{S}}_2 = t_2 \underline{\underline{S}}, \tag{9}$$

where r_1, r_2 are the strain rates, and t_1, t_2 are the stress localization factors. The macroscopic strain rate and stress state are obtained from:

$$\underline{\underline{\dot{\varepsilon}}} = f_1 \underline{\underline{\dot{\varepsilon}}}_1 + f_2 \underline{\underline{\dot{\varepsilon}}}_2, \tag{10}$$

$$\underline{\underline{S}} = f_1 \underline{\underline{S}}_1 + f_2 \underline{\underline{S}}_2. \tag{11}$$

It can be shown using the previous relations and hypotheses that the above tensorial relations can be written using only equivalent quantities:

$$\bar{\dot{\varepsilon}}_1 = r_1 \bar{\dot{\varepsilon}}, \quad \bar{\dot{\varepsilon}}_2 = r_2 \bar{\dot{\varepsilon}}, \tag{12}$$

$$\bar{\sigma}_1 = t_1 \bar{\sigma}, \quad \bar{\sigma}_2 = t_2 \bar{\sigma}, \tag{13}$$

$$\bar{\dot{\varepsilon}} = f_1 \bar{\dot{\varepsilon}}_1 + f_2 \bar{\dot{\varepsilon}}_2, \tag{14}$$

$$\bar{\sigma} = f_1 \bar{\sigma}_1 + f_2 \bar{\sigma}_2. \tag{15}$$

The r_1, r_2, t_1, t_2 quantities are not independent, the following relations are valid:

$$f_1 r_1 + f_2 r_2 = 1, \quad f_1 t_1 + f_2 t_2 = 1. \tag{16}$$

3. Effective strain rate sensitivity in strain-controlled case

One can identify the conditions of testing as *strain-controlled* if the test is conducted by prescribing the strain rate. That is, when the boundary conditions are formulated in terms of strains. In such a case, the rule of mixtures for the stresses can be a starting point (Eq. (15)). Using Eqs. (1)–(3), one obtains:

$$k\left(\frac{\bar{\dot{\varepsilon}}}{\dot{\varepsilon}_0}\right)^m = f_1 k_1 \left(\frac{\bar{\dot{\varepsilon}}_1}{\dot{\varepsilon}_0}\right)^{m_1} + f_2 k_2 \left(\frac{\bar{\dot{\varepsilon}}_2}{\dot{\varepsilon}_0}\right)^{m_2}. \tag{17}$$

In order to determine the strain rate sensitivity parameter of a material, the experimentalist applies a jump in the strain rate. Let us change the applied strain rate on the composite by a factor of α:

$$\bar{\dot{\varepsilon}} \to \alpha \bar{\dot{\varepsilon}}. \tag{18}$$

It is assumed here that α is not too large (around unity). In this first modeling, we assume that the strain rates are multiplied by the same factor in the two phases

$$\bar{\dot{\varepsilon}}_1 \to \alpha \bar{\dot{\varepsilon}}_1, \quad \bar{\dot{\varepsilon}}_2 \to \alpha \bar{\dot{\varepsilon}}_2. \tag{19}$$

Using Eqs. (18) and (19) in (17) yields:

$$k\alpha^m \left(\frac{\bar{\dot{\varepsilon}}}{\dot{\varepsilon}_0}\right)^m = f_1 k_1 \alpha^{m_1} \left(\frac{\bar{\dot{\varepsilon}}_1}{\dot{\varepsilon}_0}\right)^{m_1} + f_2 k_2 \alpha^{m_2} \left(\frac{\bar{\dot{\varepsilon}}_2}{\dot{\varepsilon}_0}\right)^{m_2}. \tag{20}$$

Another reason to keep α around unity is that – in principle – m can depend on the applied strain rate. Assuming that this dependence is negligible at this point (which will be confirmed by the experimental results below), we employ a Taylor series around $\alpha = 1$:

$$\alpha^m \cong 1 + m(\alpha - 1). \tag{21}$$

Using a Taylor series also for α^{m_1} and α^{m_2}, as well as relation (17), the effective strain rate sensitivity of the composite can be expressed from (20):

$$m = \frac{f_1 k_1 m_1 (\bar{\dot{\varepsilon}}_1/\dot{\varepsilon}_0)^{m_1} + f_2 k_2 m_2 (\bar{\dot{\varepsilon}}_2/\dot{\varepsilon}_0)^{m_2}}{f_1 k_1 (\bar{\dot{\varepsilon}}_1/\dot{\varepsilon}_0)^{m_1} + f_2 k_2 (\bar{\dot{\varepsilon}}_2/\dot{\varepsilon}_0)^{m_2}}. \tag{22}$$

Employing now the strain localization factors (Eq. (12)), a general formula for m can be obtained:

$$m = \frac{f_1 k_1 m_1 + f_2 k_2 m_2 R}{f_1 k_1 + f_2 k_2 R}, \tag{23}$$

where

$$R = \frac{r_2^{m_2}}{r_1^{m_1}} \left(\frac{\bar{\dot{\varepsilon}}}{\dot{\varepsilon}_0}\right)^{m_2 - m_1}. \tag{24}$$

Using Eqs. (4) and (5), the effective m-value can also be expressed as a function of the stress ratio t_2/t_1:

$$m = \frac{f_1 m_1 + f_2 m_2 (t_2/t_1)}{f_1 + f_2 (t_2/t_1)}. \tag{25}$$

As can be seen from this formula, the rule of mixture (Eq. (6)), usually employed in the literature to obtain the effective m-value, corresponds to the case when $t_2/t_1 = 1$ meaning equal stress states in the phases, that is, when the Static model applies. The Static model, however, cannot be used in relation with formula (22), which was obtained from the assumption that the boundary conditions are controlled in strain.

For the special case when the two phases deform identically (Taylor model) $r_1 = r_2 = 1$, thus m can be obtained from:

$$m = \frac{f_1 k_1 m_1 + f_2 k_2 m_2 (\bar{\dot{\varepsilon}}/\dot{\varepsilon}_0)^{m_2 - m_1}}{f_1 k_1 + f_2 k_2 (\bar{\dot{\varepsilon}}/\dot{\varepsilon}_0)^{m_2 - m_1}}. \tag{26}$$

4. Effective strain rate sensitivity in stress-controlled case

In contrast to the strain-controlled case, the conditions of testing can also be defined by imposing the stress state. In such a case, the boundary conditions are formulated in terms of stresses. Therefore,