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Modeling non-Newtonian slurry convection in a vertical fracture

Dmitry Eskin

DBR Technology Center, Schlumberger, 9450 - 17 Avenue, Edmonton, AB, Canada T6N 1M9

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ABSTRACT

A model of non-Newtonian slurry convection in a fracture was developed. Based on the simulation [Eskin, D., Miller, M., 2008. A model of non-Newtonian slurry flow in a fracture. Powder Technol. 182, 313–322] and experimental [Tehrani, M.A., 1996. An experimental study of particle migration in pipe flow of viscoelastic fluids. J. Rheology 40, 1057–1077] results on particle migration across a fracture, an accepted modeling system is a three-layer flow consisting of the central core of high particle concentration surrounded by pure fluid layers. The obtained solution describes convection in a small fracture domain where both the mean shear rate and the local particle concentration are known. Numerical study of the developed model shows that the solids settling rate caused by convection is much higher (regularly, by a factor of 10–30) than the particle settling rate, calculated based on an assumption that the particle concentration is uniformly distributed across a fracture. The convection model can be incorporated in one of the known numerical codes for computation of slurry dynamics in a whole fracture. An engineering modification of the convection model allows computing particle slug transport in a fracture.

1. Introduction

The hydraulic fracturing technology is an important tool of well stimulation for providing a significant increase in oil production. The principle of this technology is simple. Slurry consisting of a viscous fluid (gel) and nearly spherical particles (proppant) is pumped into a wellbore under high pressure. Slurry flows into the reservoir through perforated wellbore sections and fractures a formation. A slurry supply with a high flow rate causes fracture growth to very large dimensions. The closure pressure (external far-field confining stress applied perpendicularly to the plane of crack) tending to close a fracture is equilibrated by a pressure in a fracture that decreases from a maximum value (the pressure in the wellbore) toward the fracture tip. After a pumping stage is accomplished and pumps are stopped, the fracture is closed. During both the pumping and closure stages, a gel is filtered into formation (leakoff phenomenon). A narrow fracture channel remains open after the closure because it is filled with proppant. Very long and high fractures can be created. A fracture length may reach hundreds of meters and a fracture height may reach tens of meters. A fracture width is usually smaller than 10 mm. A fracture plays the role of a high conductivity channel because it is filled with proppant particles, the size of which are significantly bigger than the size of the particles composing the formation. A significant increase in the total reservoir conductivity due to hydraulic fracturing often causes a tremendous increase in well production. Note that hydraulic fracturing is a complicated and extensively studied technology (see, for example, Economides and Nolte, 2000). There are a number of computational codes for calculating technological parameters of a fracturing procedure. Such codes solve equations describing slurry flow in a fracture, coupled with equations of fracture mechanics formulated for a formation being fractured. Nevertheless, the percentage of fracturing procedure failures is high (above 30%). Thus, it is important to enhance the accuracy of modeling of different components of hydraulic fracturing to make this procedure a more reliable operation. In this respect, it is important to mention the paper (Pearson, 1994) in which the author presented a framework of a global model of suspension transport in a fracture. In the current work, we consider a slurry convection that is an important phenomenon affecting slurry transport. The convection occurs because of a non-uniform distribution of particle concentration across a fracture. This non-uniformity is caused by particle migration to the fracture center. A plausible explanation of this phenomenon is as follows.

In a dense slurry flow moving under a high shear rate (the mean shear rate $\dot{\gamma}_m = u_m/w$ can be as high as 200 1/s, where u_m is the superficial slurry velocity, w is the fracture width), solid particles belonging to neighboring flow layers interact with each other, which causes their fluctuation motion. The highest particle fluctuation energy is generated in the fracture wall vicinity where the shear rate is maximal. Obviously, the lowest fluctuation energy is in the fracture

E-mail address: deskin@slb.com.

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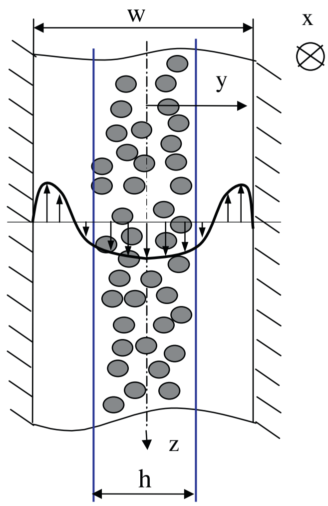

Fig. 1. Computation diagram of slurry convection.

center. Particles migrate from the area of the higher fluctuation energy to the area of the lower fluctuation energy: from the wall to the center. This process is similar to the thermal conduction in a molecular gas if particles are considered as an analogue of molecules. In such a case, the particle fluctuation energy plays the role of the kinetic energy of molecules and the particle concentration is an analog of the gas density. There are many publications on particle migration in a shear flow (for example, Nott and Brady, 1994). Eskin and Miller (2008) developed a model of particle migration across a fracture for a carrier fluid characterized by a power-law rheology. In that case, due to particle migration, the central fracture zone is characterized by practically constant particle concentration. The simulation results are in qualitative agreement with the experimental data of Tehrani (1996), who studied the particle migration in non-Newtonian slurry flowing in a pipe of 6 mm diameter. Because most fracturing fluids are characterized by power-law rheology, it is reasonable to model a flow in a fracture as consisting of the three layers. The central layer carrying particles is surrounded by layers of a pure fluid. Such a flow structure is considered in the present paper (Fig. 1). Note that the model developed in this paper can be easily extended to the central core with a variable solids concentration. In that case, a solids concentration as a function of the distance from a fracture center should be used as derivation of the model equations.

A number of investigators have studied the slurry convection. For example, Hammond (1995) and Mobbs and Hammond (2001) modeled transport of Newtonian slurry in a fracture taking convection into account. These authors assumed that particle migration leads to formation of an area with packing solids concentration in the fracture center. It was found that taking convection into account leads to a significant increase in the solids settling rate compared to the settling rate of a single particle.

The model developed in this paper can be used as a basis of a computational code describing slurry placement accompanied by convection into a fracture.

2. The model

Let us consider a fracture as a channel of the constant width. This assumption is reasonable since the fracture length and height are much bigger than its width. We will develop a model of a laminar multilayer flow based on equations of momentum and mass conservation for each layer using the lubrication approximation of a flow in a fracture (for example, Pearson, 1994). In this case, only the shear stresses caused by a friction with the fracture walls act on flowing slurry. Then the shear stress can be considered as a vector.

Note also that due to the high viscosity of a shear-thinning fluid in the central core caused by the low shear rate, the particle fluctuations in the central area are negligibly small (Eskin and Miller, 2008) and can be ignored.

2.1. Motion of pure fluid layers

The momentum equation is written as

$$\frac{\partial \bar{\tau}}{\partial y} = \bar{\nabla} p - \rho_f \bar{g} \tag{1}$$

where $\bar{\tau}$ is the shear stress, $\bar{\nabla} p = (\partial p/\partial x)\bar{i} + (\partial p/\partial z)\bar{j}$ is the pressure gradient, $\bar{i}$ and $\bar{j}$ are the unit vectors for x and z coordinates, respectively, $\bar{g}$ is the gravity acceleration, ρ_f is the fluid density.

After integration we obtain

$$\bar{\tau} = (\bar{\nabla} p - \rho_f \bar{g}) y + \bar{c}_1 \tag{2}$$

where $\bar{c}_1$ is the integration constant.

The integration constant can be determined from the momentum balance in the z direction:

$$\tau_{yz}|_{w/2} - \frac{\partial p}{\partial z}\frac{w}{2} + \rho_f g\left(\frac{w-h}{2}\right) + \rho_m g \frac{h}{2} = 0 \tag{3}$$

where h is the width of the central core; $\rho_m = \rho_s c + \rho_f(1-c)$ is the slurry density in the central core, c is the particle volume concentration in the central core, ρ_f and ρ_s are the fluid and particle densities, respectively.

Note that Eq. (3) implicitly expresses the condition of shear stress continuity on the boundary between the pure fluid layer and the central core because it does not contain any shear stress on this boundary.

The vertical component of the shear stress at the wall can also be calculated by using Eq. (2) as

$$\tau_{yz}|_{w/2} = \left(\frac{\partial p}{\partial z} - \rho_f g\right)\frac{w}{2} + c_{1z} \tag{4}$$

Substituting this equation into Eq. (3) and performing a routine math, we obtain the equation for the constant c_{1z} as

$$c_{1z} = -(\rho_m - \rho_f) g \frac{h}{2} \tag{5}$$

Then the equation for the total shear stress is written as

$$\bar{\tau} = \frac{\partial p}{\partial x}\bar{i} y + \left(\left(\frac{\partial p}{\partial z} - \rho_f g\right) y - (\rho_m - \rho_f) g \frac{h}{2}\right)\bar{j} \tag{6}$$

The absolute value of the total stress is

$$\tau = \sqrt{\left(\frac{\partial p}{\partial x}\right)^2 y^2 + \left(\left(\frac{\partial p}{\partial z} - \rho_f g\right) y - (\rho_m - \rho_f) g \frac{h}{2}\right)^2} \tag{7}$$

On the other hand, the shear stress can be written as

$$\tau = -\kappa \left(\frac{\partial u_f}{\partial y}\right)^n \tag{8}$$

where n and κ are the fluid rheology parameters.

Then Eqs. (7) and (8) allow formulating the differential equation of a pure fluid motion as

$$\frac{\partial u_f}{\partial y} = -\left(\frac{\tau}{\kappa}\right)^{1/n} = -\left(\frac{1}{\kappa}\right)^{1/n}\left(\left(\frac{\partial p}{\partial x}\right)^2 y^2 + \left(\left(\frac{\partial p}{\partial z} - \rho_f g\right) y - (\rho_m - \rho_f) g \frac{h}{2}\right)^2\right)^{1/2n} \tag{9}$$