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Elastic properties of a material composed of alternating layers of negative and positive Poisson's ratio

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ABSTRACT

The theory of elasticity predicts a variety of phenomena associated with solids that possess a negative Poisson's ratio. The fabrication of metamaterials with a '*designed*' microstructure that exhibit a Poisson's ratio approaching the thermodynamic limits of 1/2 and −1 increases the likelihood of realising these phenomena for applications. In this work, we investigate the properties of a layered composite, with alternating layers of materials with negative and positive Poisson's ratio approaching the thermodynamic limits. Using the finite element method to simulate uniaxial loading and indentation of a free standing composite, we observed an increase in the resistance to mechanical deformation above the average value of the two materials. Even though the greatest increase in stiffness is gained as the thermodynamic limits are approached, a significant amount of added stiffness can be attained, provided that the Young's modulus of the negative Poisson's ratio material is not less than that of the positive Poisson's ratio material.

1. Introduction

Poisson's ratio is defined as the ratio of the lateral strain to the longitudinal strain in a material with free boundaries which is subjected to a uniaxial stress. From the classical theory of elasticity, it is known that isotropic materials exhibit a Poisson's ratio within the range $-1 \leq \nu \leq +0.5$ [1]. Most materials exhibit a Poisson's ratio that is positive; when stretched the material becomes thinner and when compressed it becomes wider. Materials with a negative Poisson's ratio now known as "auxetic" materials, exhibit the opposite behavior. Researchers have postulated that such counterintuitive behavior could be beneficial [2–4]. Experimental confirmation of the expected behavior has been hindered by the fact that materials exhibiting a negative Poisson's ratio are rare, and when they do occur, they show only small negative values.

There are naturally occurring auxetic materials, such as the nitrides [2], metals [3], zeolites [4], and silicates [5,6] as well as man-made examples, such as the re-entrant foams [7], nanostructured [8] and microporous [7,9] materials. The recent demonstration that metamaterials can be fabricated with a desired microstructure (nanostructure) of elements that are connected so that the material possesses a large negative value of Poisson's ratio [7] has raised the possibility of using auxetic materials to produce composites with desirable bulk properties. One of the first reports of the production of such a composite material was a report of auxetic foam in 1987; foam samples were produced from conventional foams using a compression-heating process [7]. Auxetic materials are now used in composite materials to promote fibre reinforcement, in textiles for crash helmet and sports clothing manufacture, as sponges, as ropes, in filtration, as shock absorbing materials, and in biomedical applications. In fibre composites, fibre pull-out is a major failure mechanism. When a unidirectional composite is loaded in tension both the matrix and fibre undergo lateral contraction, which leads to failure at the fibre–matrix interface. In contrast, when using auxetic fibres the interface is maintained by matching the Poisson's ratios of the matrix and fibre; such that the auxetic fibre expands by the same amount that the matrix contracts minimizing the interface lateral stresses. This feature, in combination with enhanced stiffness, could enable fibres optimised for use in composites to be produced.

In this paper, we study the possibility of constructing a composite layered material that exhibits exceptional elastic properties in particular, a significant increase in the stiffness. The approach we use is to investigate the mechanical behavior of a multilayered system using the finite element method. We assume no constraints apply to the Poisson's ratio other than the theoretical thermodynamic limits and explore the increases that are possible in the Young's modulus of a multilayer system in which positive and negative Poisson's ratio materials alternate. The aim is to quantify any benefits to the stiffness of a material that can be gained.

2. The finite element method

In this work the finite element package STRAND7 [10] was employed. The finite element method has no restrictions on the geometry of the system, and permits the use of any boundary con-

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Table 1
The normalised effective Young's modulus for an increasing number of layers. Column A shows the result for the sequences shown and column B shows the result for the inverse sequences in which positive and negative values of Poisson's ratio are interchanged.

Model height (mm)	Layer type and order (positive Poisson's ratio (+), negative Poisson's ratio (−))	Normalised effective Young's modulus	
		A	*B*
20	+, −	2.90	2.90
30	+, −, +	2.01	3.60
40	+, −, +, −	2.96	2.96
50	+, −, +, −, +	2.38	3.57
60	+, −, +, −, +, −	2.98	2.98
70	+, −, +, −, +, −, +	2.56	3.46
80	+, −, +, −, +, −, +, −	3.00	3.00
90	+, −, +, −, +, −, +, −, +	2.67	3.38
100	+, −, +, −, +, −, +, −, +, −	3.01	3.01
110	+, −, +, −, +, −, +, −, +, −, +	2.74	3.33
120	+, −, +, −, +, −, +, −, +, −, +, −	3.02	3.02
130	+, −, +, −, +, −, +, −, +, −, +, −, +	2.79	3.29
140	+, −, +, −, +, −, +, −, +, −, +, −, +, −	3.03	3.03
150	+, −, +, −, +, −, +,−, +, −, +, −, +, −, +	2.83	3.27
190	+, −, +, −, +, −, +, −, +,−, +, −, +, −, +, −, +, −, +, . . .	2.88	3.23
230	+, −, +,−, +, −, +, −, +, −, +, −, +, −, +, −, +, −, +, −, +, −, +, . . .	2.91	3.20

ditions. The finite element model was constructed as a free standing multilayered structure consisting of alternating layers of two materials. To represent a uniaxial load, the boundary conditions were applied so that the top surface is constrained to remain flat but can move freely in the *z* direction and freely slip in the *x*–*y* plane. The bottom surface is fixed in the *z* direction but can freely slip in the *x*–*y* plane. The convergence of the finite element results were all confirmed with respect to the element size and shape. To simplify the calculations, the interfaces between layers were assumed to be perfectly bonded, and thus, failure along the interface (or any other point in the structure) was not modeled.

For small applied uniaxial loads, the effective Young's modulus of the structure was determined for various structural configurations. The effective Young's modulus of the structure is a function of the Young's modulus of the constituent layers and scales linearly with the magnitude of the Young's modulus of the layers. For convenience, the Young's modulus of both types of layer was preset to the same value. The result for the effective Young's modulus was normalised with respect to the Young's modulus of the layers to give a result independent of the choice of the Young's modulus of the layers.

To characterize the mechanical behavior of the configuration, the layer thickness ratio (i.e. the ratio of the thickness of the negative Poisson's ratio layers to the positive ones), the number of alternating layers, the model dimensions and the elastic properties of the layers were all systematically varied. To determine the effective Young's modulus of the composite layered structure, a finite element model with dimensions of 200 mm × 200 mm in the *x*–*y* plane and 10 mm thick layers, was constructed.

3. Results and discussion

The number of layers was increased and the effective Young's modulus of the structure was calculated as a function of the number of layers. The order of the layers was also changed. The Poisson's ratio of the negative and positive layers was set at −0.9 and +0.4, respectively. The aspect ratio of the layers, defined as the ratio of the layer width to layer thickness, was fixed at 20. The results of these calculations are presented in Table 1. This data is also presented as a plot in Fig. 1. The results for the normalised effective Young's modulus approach the same value for a large number of layers. The highest effective Young's modulus is obtained from the

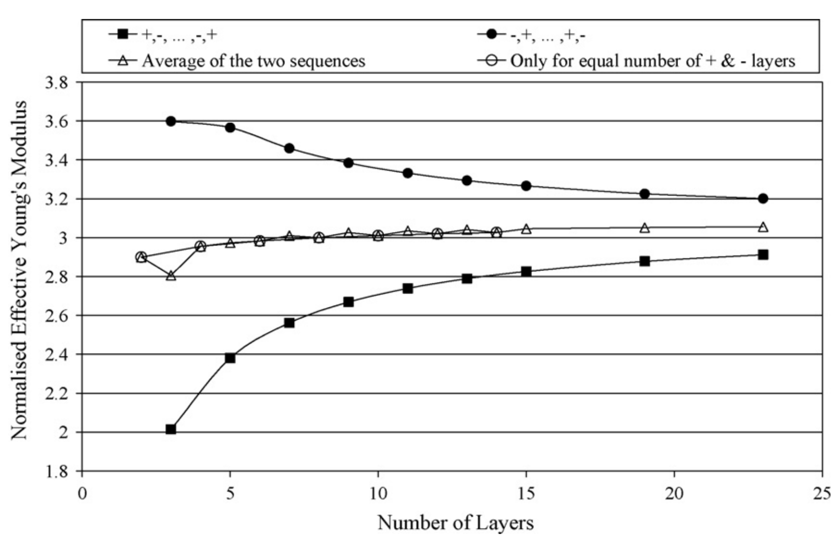

Fig. 1. The normalised effective Young's modulus as a function of the number of layers. The aspect ratio of the layer width to height is 20. The number of layers is an odd number for the upper (solid circle) and lower (solid square) curves. The upper curve is the case where the first and last layers have a negative Poisson's ratio. The lower curve is the case where the first and last layers have a positive Poisson's ratio. The open circles are for the calculation for a structure with an even number of layers. The average of the upper and lower curves (open triangles) coincides with the result of the even number of layers, except for the three layer case.