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An analytical two-temperature model for convection–diffusion in multilayered systems: Application to the thermal characterization of microchannel reactors

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Abstract

The main purpose of this paper is to implement some convenient analytical solutions of the two-dimensional convection–diffusion equations in a multilayered system, in the form of some relationships between average temperature fields, based on the thermal quadrupole formalism. Some equivalent analogical networks are proposed in order to implement the model in a more convenient form, based on the electrical analogy. The important advantage of such approach is to connect different layers through simple network connections between the respective interface variables. Special emphasis is laid on the case where the lateral boundary conditions correspond to insulated walls. The transient case is also presented, for non-insulated lateral boundary conditions, coupled with a third layer. Some examples are given in order to illustrate the suitability of the proposed model in the case of temperature field image processing in a microfluidic chip.

Keywords: Heat conduction; Two-temperature model; Microfluidics; Porous media; Thermal quadrupole; Temperature field processing

1. Introduction

The increasing development of microfluidic chemical reactors is due to its ability to create intensive continuous reactive rates involving small amounts of samples (Salmon et al., 2005). The thermal characterization of microchannel reactors is an important requirement, and can be a promising way for retrieving the reaction dynamics from the analysis of the thermal sources distribution. While the measurement of the absolute local temperatures is a quite difficult problem (Möllmann et al., 2004), an alternative method for the thermal characterization is to retrieve the source term distribution by processing the relative temperature field on the whole microfluidic chip (Pradère et al., 2006). However, processing this kind of local information is a quite ill-posed problem, due to the low signal/noise ratio and the fin effect within the closing substrate (see Fig. 1 where a microfluidic chip is shown). It is then of interest to estimate the thermal parameters from a macroscopic averaged approach, where the thermal field is averaged in the lateral direction perpendicular to the microchannel within the plane of the chip. Averaging should yield low-pass filtering, allowing to estimate heat losses through the closing substrate wall. Also averaging the temperature field yields the possibility of processing the data with a low spatial resolution, when the microchannel is almost not visible, as will be shown in the last example, proposed in Section 5 of the present paper.

The two-temperature models proposed herein are used to describe the evolution of the average temperature and total heat flux along the microchannel as a function of the heat source distribution within a microchannel reactor. This kind of models are of particular interest for the thermal characterization of fluid flow parameters in microchannel reactors, where both the chemical reaction heat source and the convective heat transfer coefficients could be estimated from global measurements and

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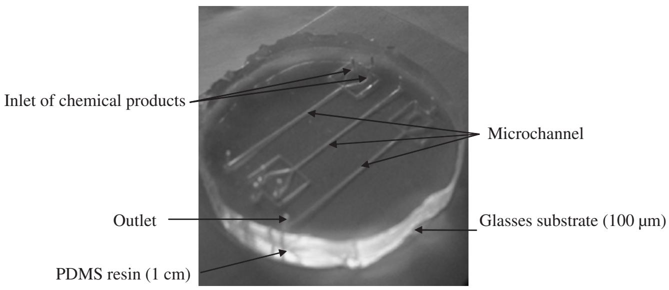

Fig. 1. Microfluidic chip.

inverse methods. As the size of a channel is reduced, the surface to volume ratio physical effects tend to grow up, and the continuum flow assumptions may be turned out. An intermediate situation is when the usual continuum equations still apply, but with some modifications on the boundary conditions (slip flow). Heat transfer in rectangular microchannels has been addressed by many authors in that intermediate case, in order to define some convenient correlations involving slip coefficient and Nusselt number (Tunc and Bayazitoglu, 2002). Peng and Peterson (1995) studied the effect of thermofluid and geometrical parameters on convection of liquids through rectangular microchannels. This kind of approach is not within the scope of the present paper. However, even for relatively large micro/minichannels, the large surface to volume ratio implies a quite multidimensional heat transfer behavior, while the coupling effects between the walls and fluid bulk temperatures are turned out into an important factor, as pointed out by Maranzana et al. (2004). Microchannels are often used as microreactors, and various recent studies deal with the corresponding microcalorimetric measurements (Zhang and Tadigadapa, 2004). For instance, van Male et al. (2004) analyze heat and mass transfer in a square microchannel heated from one side, and establish the corresponding correlations for the Nusselt and Sherwood numbers as a function of the Graetz number.

The main purpose of this paper is to implement some convenient analytical solutions of the two-dimensional convection–diffusion equations in a multilayered system, in the form of some relationships between average temperature fields. The model is based on the thermal quadrupole formalism (Maillet et al., 2000). The basic thermal quadrupole formalism is a very efficient method for linear diffusion modeling, when involved in multilayered systems. For homogeneous media, a linear intrinsic transfer matrix relates the input/output temperature and heat flux after a Laplace transformation and some convenient integral space transforms (Batsale et al., 1994), yielding some exact analytical relationships in the transformed space. The main advantage of this kind of formulation is to make easy the simple representation of multilayered systems by just multiplying the corresponding quadrupole matrices. More recently, this formalism gained a generalized semi-analytical approach for heterogeneous stratified media (Fudym et al., 2002), allowing to split the conductive heat transfer into an homogenized transfer in series with a constriction term, so that a conductive boundary layer was defined (Fudym et al., 2004).

Some equivalent analogical networks are proposed in order to implement the model in a more convenient form, based on the electrical analogy. Some examples are given in order to validate and to illustrate the suitability of the proposed model for the image processing of temperature fields in a microfluidic chip.

2. Statement of the problem and solution methodology

The problem of interest in this paper, as shown in Fig. 2, is given for a two-layer medium in transient state with source term and constant velocity. The different applications envisioned in the next sections will contribute to illustrate what media are in consideration herein. For instance, medium 1 is a microchannel with reactive fluid flow, and medium 2 is the closing substrate. Or medium 1 is a heating porous material, while medium 2 is the gas flow in the void fraction. The thermal behavior of this system is described by the following governing equations and boundary conditions:

$$\frac{\partial^2 T_i}{\partial x^2}+\frac{\partial^2 T_i}{\partial y^2}+\frac{g_i(y,t)}{k_i} = \frac{1}{a_i}\frac{\partial T_i}{\partial t}+\frac{v_i}{a_i}\frac{\partial T_i}{\partial y}, \quad i=1,2, \tag{1a}$$

$$x=0, \quad -k_1\frac{\partial T_1}{\partial x}=\varphi_0(y,t), \tag{1b}$$

$$x=e_1, \quad T_1=T_2 \quad \text{and} \quad k_1\frac{\partial T_1}{\partial x}=k_2\frac{\partial T_2}{\partial x}, \tag{1c}$$

$$x=e_1+e_2 - k_2\frac{\partial T_2}{\partial x}=\varphi_s(y,t), \tag{1d}$$

$$t=0, \quad T_1=T_2=0. \tag{1e}$$

The boundary conditions relative to $y=0$ and $y=L$ are not given specifically herein, but are assumed to be linear and homogeneous, and given by some relationships between the corresponding temperature and flux. These conditions will be specified according to the different cases envisioned hereafter. The sources and boundary conditions relative to y are assumed to be independent of x.

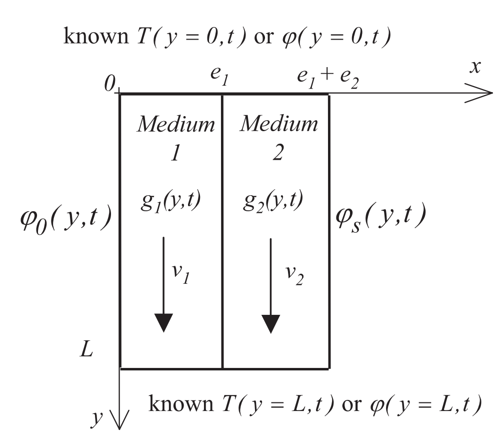

Fig. 2. Geometry and boundary conditions of the two-layer problem.