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On the modeling of shape memory alloys using tensorial internal variables

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Abstract

In the literature, some experimental results of polycrystalline NiTi shape memory alloys reveal a strong change in temperature occurring already in a strain range which is commonly regarded as linear elastic. This is rather surprising, since it may be an indication for an evolution of martensite in the linear elastic region. To consider this effect in material models, a tensorial internal variable for the description of the phase transformation along with a suitable approach for the inelastic strain rates resulting from the formation of martensite may be used. Stress-induced phase transformations within shape memory alloys can be regarded as straightened transformations, i.e. during a forward transformation, martensite variants which fit best to the current stress state are favored. Due to this, an appropriate description for the evolution of phase transformations should provide two information: first, the scalar mass fraction of martensite, and second, the orientation of the martensite variants. The internal tensorial variable which is introduced in this article is based on the growth of one single martensite variant. From this variable the information about the mass fraction of martensite and about the average direction of the martensite variants can be extracted. Exemplarily, the embedded tensorial variable within the material model developed by Raniecki et al. [B. Raniecki, C. Lexcellent, K. Tanaka, Arch. Mech. 44 (3) (1992) 261–284] and extended by Müller [C. Müller, Thermodynamic Modeling of Polycrystalline Shape Memory Alloys at Finite Strains, Mitteilungen aus dem Institut für Mechanik, Ruhr-Universität Bochum, 2003] is discussed.

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1. Introduction

Martensitic phase transformations for which the variants of the developing phase prefer a distinct orientation can be regarded as straightened transformations. Among the 24 possible martensite variants which may form during a temperature-induced phase transformation only those are preferred during a stress-induced phase transformation which accommodate the stress state best (see e.g. [1]). This leads to a straightened martensitic structure and as a consequence, to a macroscopic deformation. Thus, the martensite within this structure can be regarded as oriented martensite. Since the macroscopic deformation vanishes upon unloading, it is referred to as pseudoelastic deformation. In contrast to this, the martensitic structure which forms during a temperature-induced phase transformation does not exhibit any distinct orientation. Hence, no macroscopic strain can be observed, so that this kind of transformation may be regarded as unstraightened transformation. The resulting martensite is referred to as self-accommodated martensite, since the martensite variants accommodate structural shape changes resulting from the phase transformation (cf. [2]).

The appropriate description of self-accommodated and oriented martensite may prove crucial for the development of a general framework for a macroscopic continuum material model, since the phase transformation is the basic property of shape memory alloys. On the one hand self-accommodated and oriented martensite occur at pseudoplasticity. There, during loading, self-accommodated martensite is deformed elastically before undergoing a stress-induced reorientation, leading to oriented martensite and in turn to a high inelastic strain. On the other hand, self-accommodated and oriented martensite may also occur at pseudoelasticity. Since, to the knowledge of the authors, the latter has not been extensively discussed in the lit-

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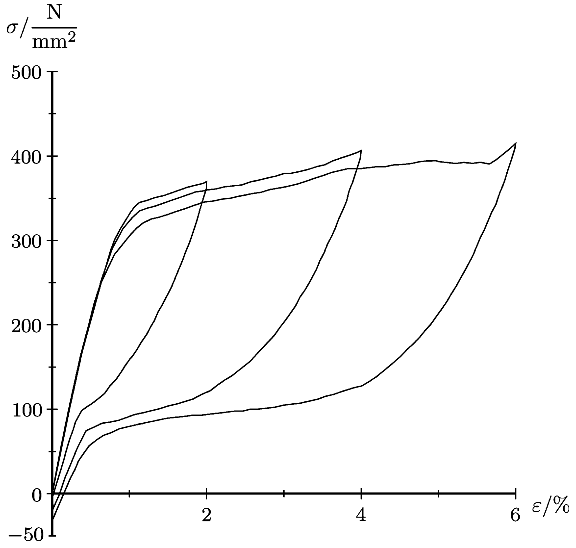

Fig. 1. Stress–strain curves of tests in simple tension for maximum strains of 2, 4, and 6% at strain rate of 10^{-3} (see [3]).

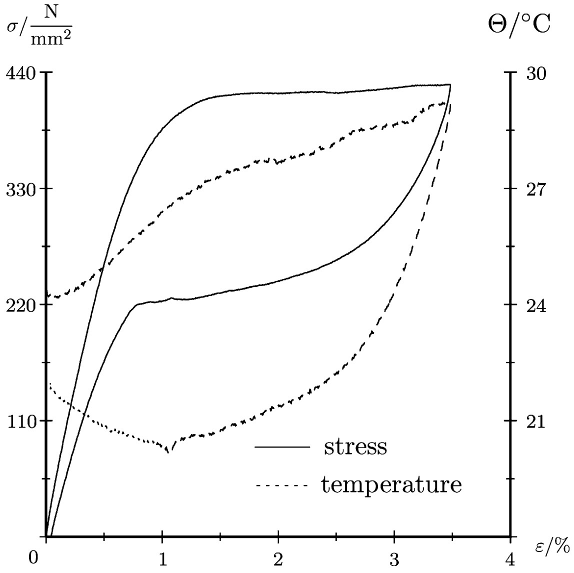

Fig. 3. Stress–strain and temperature–strain curves of a test in simple tension for maximum strain of 3% at strain rate of 10^{-4} (see [4] for experimental setup).

erature, it will be analyzed here more in detail. In Figs. 1–3 , the temperature evolutions during tests in simple tension of pseudoelastic, polycrystalline NiTi shape memory specimens are depicted. The corresponding experiments to Figs. 1 and 2 were conducted by Helm (see [3] for experimental setup and preparation of the sample) and the results in Fig. 3 were obtained by Grabe and Bruhns (see [4] for details concerning test program and sample preparation). First consider the stress–strain diagrams in Figs. 1 and 3. There, a typical behavior of a pseudoelastic material can be seen. On the loading path, following the common theory, a region where the austenite is deformed nearly linear elastically is followed by a region of high inelastic strain at approximately constant stress. The inelastic strain has its origin in the formation of martensite during the forward phase transformation. The unloading path can be described in an analogous manner: the nearly linear elastic deformation of the martensitic phase is followed by the inelastic strain due to the reverse phase transformation first, and finally by the elastic deformation of the austenite. Now, considering the temperature evolution, the common theory reveals a contradiction. Generally, for a purely elastic test in tension, the temperature of a specimen remains constant, or more specifically, drops slightly for loading and increases slightly for unloading due to the piezocaloric effect. Hence it is surprising, that the temperature changes all over the mechanically linear elastic regions of the martensitic and austenitic phases as in Figs. 2 and 3. Even more surprisingly, the temperature change in the mechanically linear elastic region is stronger than in the inelastic region. A possible explanation might be, that already in mechanically linear elastic regions phase transformations take place. This is based on the fact, that the austenite to martensite transformation on the loading path is an exothermic whereas the martensite to austenite transformation on the unloading path is an endothermic process. Within this context, two different kinds of austenite to martensite transformations might be possible for the loading path: first, it could be an austenite to R-phase transformation present, and second, the transformation could proceed from austenite to self-accommodated martensite of the B 19′phase. Results of a differential scanning calorimetry (DSC) measurement of the sample analyzed in Fig. 3 are shown in Fig. 4. On the cooling path, a peak of a multistage transformation (see 1) is followed by one single peak (see 2). Furthermore, the height of the multistage transformation peak is by a magnitude smaller than the height of the following single peak. The multistage transformation peak may have its origin in an austenite to R-phase transformation. However, as discussed in [5], the height of this peak may indicate that only the boundaries of the austenite grains transform into the R-phase, while the inside of the austenite grains stay austenitic. Thus, the influence of the R-phase on mechanical properties of the sample may be of minor importance. This supposition may be supported by the following observation: the temperature during the experimental test is

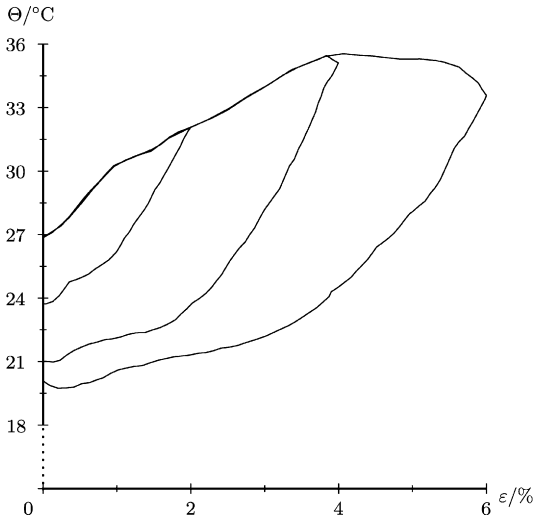

Fig. 2. Corresponding temperature evolution to Fig. 1.