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Topological hysteresis as a model for Rayleigh damping

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Abstract

To account for the large magnetomechanical damping of demagnetized specimens, we propose as a model Landau's arrangement of magnetic domains with no overall magnetic moment. To analyze the model, we use the nature of magnetostriction to transform the data from internal friction, Q^{-1}, and strain amplitude, ε, into energy loss and volume fraction reoriented by stress, both as functions of ε. The large peak in $Q^{-1}(\varepsilon)$ above the upper end of the Rayleigh range is attributed to the onset of gross changes in the domain structure, the Rayleigh range to dynamics of closure domains on a finer scale, and the lower end of the range to the wall displacements becoming of the order of one atomic distance.

Keywords: Magnetomechanical damping; Hysteresis; Magnetic domains; Domain wall motion

1. Introduction

Magnetomechanical damping (MMD) has marked parallels with magnetic hysteresis (MH) suggesting that the mechanisms are essentially the same, that the damping is a form of ferromagnetic hysteresis [1–3]. However, neither has been satisfactorily explained in terms of the domain model of ferromagnetism. Theories based on the motion of a domain wall through a statistical distribution of obstacles have been summarized recently [4,5]. We follow Molho et al. [6] in turning instead to topological hysteresis because it presupposes only the presence of domains in a ferromagnetic material. In this paper we focus on the Rayleigh range of hysteresis in bulk, polycrystalline specimens of iron and its relatively dilute alloys, but the model we develop should have broader applicability.

From the experiments on MMD in ferrous materials, we select three noteworthy points: (i) the damping, Q^{-1}, measured in the Rayleigh range of strain amplitude ε ($10^{-6} < \varepsilon < 10^{-4}$) is largest, broadly speaking, when the overall magnetic moment $\mathbf{M} \approx 0$; (ii) for well-demagnetized specimens the oscillatory flux during vibrations is very small, so $\mathrm{d}\mathbf{M}/\mathrm{d}t = (\mathrm{d}\mathbf{M}/\mathrm{d}\sigma)(\mathrm{d}\sigma/\mathrm{d}t) \approx 0$, where t is the time and σ is the stress; (iii) Q^{-1} is largest when the composition is such that there is no γ phase at high temperature, permitting annealing at higher temperatures. Point (i) is supported by all the evidence such as [7]. Point (ii) emerges from the data of Coronel et al. [8], who observed a vibrating flux density with amplitudes as low as 1.5×10^{-8} T. Point (iii) appears from the data of Smith and Birchak (SB), summarized in [9], on the Rayleigh slope, $\theta_R = \mathrm{d}Q^{-1}/\mathrm{d}\varepsilon$, of Fe–Si and Fe–Ge alloys. SB, who actually report values of $\psi = 2\pi Q^{-1}$, find values of θ_R about 2400 for iron with 2.35% Si and 3.1% Si after annealing at 1200 °C, compared with values as low as 35 for the latter annealed at 750 °C, and 25 found by [7] for Armco iron annealed at 700 °C.

We need then a model for MMD that will give large damping in a demagnetized specimen, the damping to arise from domain walls which move in such a way that little net magnetic moment is generated during the vibrations, implying that the motion of the walls must be fairly strongly correlated.

2. The model: a Landau block

Perhaps the simplest of candidates [10] is Landau's arrangement of a group, a block, of domains in a rectangular solid with $\mathbf{M} = 0$, as shown in Fig. 1. For each boundary in Fig. 1, the component of magnetization $\mathbf{I}$ normal to the boundary is continuous across the boundary and no poles are formed ($\nabla \cdot \mathbf{I} = 0$). Fig. 1 is suitable for a cubic material with positive anisotropy such as iron, with the crystalline axes aligned with the coordinate axes; for simplicity, we will follow this case. In the absence of a field the magnetization $\mathbf{I}$ lies along a <1 0 0> direction. Two of many possible arrangements are shown in Fig. 1(a) and (b), with differing numbers N, each satisfying (i). Because of mag-

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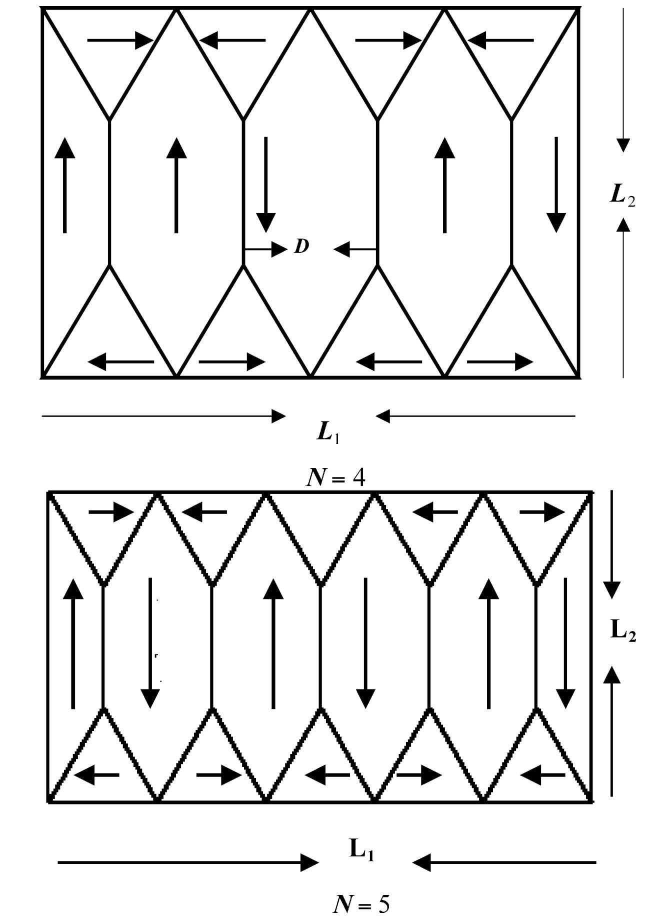

Fig. 1. Two versions of a Landau block, with domain walls seen edge on as traces. The long domains with pointed ends, in which the magnetization **I** points alternately up or down (±*y*-axis), are separated by 180° walls with traces along the *y*-axis. There are also closure domains, triangles with **I** pointing alternately right and left (±*x*-axis), separated from the long domains by 90° walls. The 90° walls always meet the outer boundary at an angle of 45°, and each other at 90° at the triple junctions with the 180° walls. Magnetostrictive strain is omitted in this description.

netostriction, an applied stress may exert a force on a 90° wall, but never on a 180° wall. Nevertheless, because of the need to preserve $\nabla \cdot \mathbf{I} = 0$ in the vicinity of the walls, especially near the triple junctions, we envision that all the domain walls, both 90° and 180° walls, move together in response to a stress. Such a motion involves states with non-integer N. However, if the angles between the walls remain constant, the pattern can still change by changing its scale in such a way that deviations from the pattern occur only at the edges where closure domains on a finer scale may appear and give rise to a small, transitory $\mathbf{M}(t)$, thus (possibly) satisfying (ii). In that case the motion of the walls is indeed highly correlated. Using this geometric model, together with the general principles of internal friction and the special assumption that the Rayleigh law holds, we can produce some useful estimates concerning the motion of the network of walls in a Landau block.

In Fig. 1 we see a cross-section through a structure that does not vary in the third direction, so the areas seen are in proportion to the volumes of the domains; the area fraction of closure domains in the plane is equal to their volume fraction. The block, a rectangular solid of dimensions L_1, L_2, L_3, is mostly taken up by domains with magnetization $\mathbf{I}$ along the ±y-axis having the shape of pointed rectangles. From tip to tip the long domains measure L_2, but their width, D, is variable. The closure domains each have a base of D and an altitude of $D/2$. The structures shown are characterized by an integer, $N = L_1/D$, the number of closure domains at one end, equal to the number of 180° walls.

In analyzing the structure of Fig. 1, we must consider both the areas in the plane of the figure, A_l of the long domains and A_c of the closure domains, which represent the content of the domains, and also the areas of the walls, A_{90} and A_{180}, which are given by lengths in Fig. 1 multiplied by their extension in the perpendicular direction L_3. The length of the zigzag border of the closure domains at one end of the block is just $\sqrt{2}L_1$, while the height of any of the closure triangles is $D/2$. The total area of the closure domains, taking account of both ends, is just $A_c = DL_1/2 = L_1^2/(2N)$, and so $A_l = L_1L_2 - A_c = L_1L_2 - L_1^2/(2N)$. The areas of the walls are $A_{180} = N(L_2 - D)L_3$ and $A_{90} = 2\sqrt{2}L_1L_3$. The total domain wall energy of the block is then

$$E_{dw} = E_{90} + E_{180} = \gamma_{90}A_{90} + \gamma_{180}A_{180} = (2\sqrt{2}\gamma_{90} - \gamma_{180})L_1L_3 + \gamma_{180}NL_2L_3, \quad (1)$$

$$\Delta E_{dw} = \gamma_{180}L_2L_3\Delta N, \quad (2)$$

where γ_{90} and γ_{180} are the surface energies of the corresponding domain walls. It is striking that while the mechanical force is exerted on the 90° walls, the resulting changes of energy are stored in the 180° walls; in this model the usual neglect of interactions between the two kinds of wall is seen to miss the essence.

The magnetostrictive strain has a constant magnitude λ, more precisely λ_{100}, but may extend along any <1 0 0> direction. When there are only two <1 0 0> axes to consider, we may express this strain in terms of the volume fraction f of the closure domains. The average strains in the block of Fig. 1 are

$$\varepsilon_{11} = \lambda f, \qquad \varepsilon_{22} = \lambda(1 - f), \quad \text{all other } \varepsilon_{ij} = 0. \quad (3)$$

This result depends only on the restriction to two domain orientations and not on which particular Landau array is under consideration.

The volume fractions being equal to the area fractions, we have

$$f = \frac{A_c}{L_1L_2} = \frac{D}{2L_2} = \frac{L_1}{2NL_2}, \qquad \Delta f = \frac{\Delta D}{2L_2}. \quad (4)$$

The work done on the block by a stress σ_{11} acting on a change of strain, $\Delta\varepsilon_{11}$, is

$$\Delta W_\sigma = L_1L_2L_3\sigma_{11}\Delta\varepsilon_{11} = L_1L_2L_3\sigma_{11}\lambda\Delta f, \quad (5)$$

where a positive $\Delta\varepsilon_{11}$ corresponds to an increase in D and in the volume of the closure domains, and so to a decrease in N. The work done to bring about a change in the domain configuration of the specimen is assumed to be lost from the driving mechanical system and so to be part of the magnetomechanical damping.