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Contact analysis of fractal surfaces in earlier stage of resistance spot welding

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Abstract

A revised fractal contact model followed Zhu is utilized to analyze the contact behaviors of electrode/workpiece (E/W) and workpiece/workpiece (W/W) interfaces for aluminum alloy and low carbon steel, respectively. The influences of load, fractal parameters and temperature on the contact rate and the effect of strain hardening exponent on the plastic area fraction are explored, respectively. Analysis results show that the contact rate of W/W interface is bigger than that of E/W interface for Al and the opposite is true for Fe. For a same interface, the contact rate for Al is bigger than that for Fe. The reduction of elastic modulus will increase the contact rate. The deformation type of the contact spot under heavy load changes from elastic to plastic and again to elastic via the elastoplastic deformation. Comparison with the predicted and the experimental curve illustrates that the predicted curve is in better agreement with the measured and the analysis results are right.

Keywords: Fractal surface; Contact; Resistance spot welding

1. Introduction

Contact is a kind of universal phenomena in many engineering fields such as friction, wear, sealing, contact resistance, etc. The contact behavior between two approaching surfaces influences greatly product performance. In the early stage of resistance spot welding, the contact behavior between two workpieces or between the electrode and the workpiece determines the magnitude, distribution and heat-producing of electrical contact resistance, which has an effect on the formation of nugget, the surface sputtering and the wear of the electrodes [1,2]. To obtain proper nugget, to control the surface quality of weldments, to avoid or decrease the degree of wear, it is necessary to study the contact condition between two surfaces.

As far as contact analysis is concerned, many literatures yield valuable results [3–8]. For example, Greenwood and Williamson [3] established the criterion for an asperity deformation by assuming that the asperities heights follow Gaussian distribution, the radii of curvature are the same and the lateral distribution of the asperities are invariant. But the contact result given in this study is instrument resolution-dependent and therefore non-unique. The introduction of fractal theory and fractal geometry into the contact field solved the problem that the characterization parameters are scale-dependent [4]. Based on two-dimensional Weierstrass–Mandelbrot (*W–M*) fractal function, a contact model is developed [5], in which the asperities are either in elastic deformation or in plastic deformation, however the elastic–plastic deformation which is the transition condition from elastic deform to plastic deformation is neglected. Later, a elastic–plastic contact analysis between two approaching surfaces which are characterized by the three-dimensional Weierstrass–Mandelbrot fractal function is presented [6]. In another literature, the evolution of elastic, elastic–plastic and fully plastic deformation of layered media surface is obtained using the scale-independent fractal characterization parameters and the finite element method [7]. In the above two investigations, the elastic–plastic deformation is involved in the contact analysis, but the work hardening due to the transition from elastic condition to plastic condition is ignored. Later, the effect of work hardening is taken into account in his friction contact model by Zhu et al. [8]. But it is to be regretted that all the contact analyses are aimed at the contact problems in the fields except resistance spot welding, no open literature about the contact from fractal perspective in resistance spot welding is published as yet.

In resistance spot welding, what is the history of asperities deformation under the applied load in the earlier stage of spot

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welding, how the microcontacts distribute, what is the effect of work hardening on contact history, how the surface topography, material properties, the load applied by the electrode and the temperature influence the real contact area, what is the difference of contact behavior between the interface of E/W and that of W/W, what is the difference between spot welding of low carbon steel and that of aluminum alloys from contact perspective, all of the above problems are worth revealing. They are also the main objectives of the present article. To solve these problems, the two-dimensional Weierstrass–Mandelbrot fractal function is first introduced briefly. Because of different factors considered in the contact analysis aimed at resistance spot welding compared to the contact analysis in other fields, based on the model by Zhu, the contact model in which the work hardening is considered applied to resistance spot welding is given, finally the contact analysis correlated with resistance spot welding is conducted.

2. Fractal characterization of rough surface

It has been shown that many engineering surfaces topographies are multiscale, random and exhibit the character of self-similarity or self-affinity, which indicates that more and more details appear when the original surface topography is magnified but also the magnified topography are similar statistically to the original [4,9]. In addition, the profile image of such a surface is always continuous everywhere but non-differential when it is magnified. It is illustrated that these features of the surface topography can be satisfied by Weierstrass–Mandelbrot fractal function and the *W–M* function can be used to generate the surface topography which is the representative of the actual surface.

In the present article, the isotropic and homogeneous surface is considered. The isotropy and homogeneity of the surface mean that the profile produced along a straight line and in an arbitrary direction of the surface can be used for the valid representation of the surface [9]. The fractal dimension D_s of the surface and that D of the corresponding profile of the surface satisfy the relationship $D = D_s - 1$ [10]. The modified two-dimensional *W–M* function with fractal dimension D is given as

$$z(x) = G^{(D-1)} \sum_{n=n_1}^{\infty} \frac{\cos 2\pi\gamma^n x}{\gamma^{(2-D)n}}; \quad 1 < D < 2; \quad \gamma > 1 \tag{1}$$

where G is a characteristic length scale, $\gamma^{n_1} = 1/L$, where L is the sampling length. To provide both the phase randomization and high spectral density [4], γ is selected to be 1.5. When L is determined, the parameters G and D form the set to characterize the profile $z(x)$. The parameters of G and D can be obtained from power spectrum or structure function of the *W–M* function.

The power spectrum of *W–M* function is given as [11]

$$S(\omega) = \frac{G^{2(D-1)}}{2\ln\gamma} \cdot \frac{1}{\omega^{(5-2D)}} \tag{2}$$

where $S(\omega)$ is the power of the spectrum, ω is the frequency which is the reciprocal of the wavelength of roughness. Eq. (2) shows that the power spectrum exhibits power law behavior and the parameters of G and D are independent of the frequency ω, consequently they are scale-independent and suitable to characterize the feature of the multiscale of actual rough surface. The relationship of $S(\omega)$ and ω on the log–log plot would result in a straight line, the slope of the straight line determined the magnitude of fractal dimension D of the surface profile and the location of the straight line on the $\log S(\omega)$ axis is related to the value of the characteristic length G.

In resistance spot welding, the electrode is machined by fine turning and the workpieces regardless of low carbon steel or aluminum alloy sheets by milling, it is shown that the profiles of these surfaces exhibit fractal behavior, and can be simulated by the *W–M* function.

3. Contact mechanics and contact model

The contact of two approaching rough surfaces can be represented by the contact of a smooth rigid surface and an equivalent deformable rough surface with a reduced elastic modulus $E^* = [(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2]^{-1}$, where ν_1, ν_2 and E_1, E_2 are the Poisson's ratios and Young's moduli of the two original surfaces, respectively, the power spectrum of the equivalent surface is the sum of the power spectra of original surfaces [12]. The contact of two rough surfaces will produce numerous circular asperities microcontacts, for simplicity, in the contact analysis for resistance spot welding, the interactions of the asperities are neglected, but the work hardening is considered. To determine the real contact area, the total load carried by the microcontacts, the deformation model of an individual asperity must be established.

3.1. Elastic, elastic–plastic and plastic regimes of microcontact

Consider a circular contact spot with area a and radius of curvature R. It has been shown that when $a > a_{ec}$, where a_{ec} is the critical elastic area, the contact spot will deform elastically. The critical elastic area is given by

$$a_{ec} = G^2 \cdot \left[\frac{1}{\pi}\left(\frac{4E^*}{3k\sigma_s}\right)^2\right]^{1/(D-1)} \tag{3}$$

where σ_s is the yield strength of the softer material, for the initial yielding of the asperity, $k = 1.1$, while for the fully plastic yielding, $k \doteq 3$.

When the area of contact spot is smaller than the critical plastic area a_{pc}, the asperity will deform plastically [5,8], whereas those contact spots whose areas are in the range of a_{pc} and a_{ec} will be in elastoplastic deformation [8]. For the work hardened materials whose stress–strain relationship satisfy

$$\sigma_f = B\varepsilon^{1/m} \quad (m > 1) \tag{4}$$

where σ_f is the plastic flow stress, m the work hardening exponent, ε the total strain and B is constant, the expression of the critical plastic contact area for the inception of plasticity is given