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On the nonlinear elastic properties of textile reinforced concrete under tensile loading including damage and cracking

Mike Richter *, Bernd W. Zastrau

Institute of Mechanics and Shell Structures, Faculty of Civil Engineering, Technische Universität Dresden, D-01062 Dresden, Germany

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Abstract

This paper is focussed on the description of the macroscopic nonlinear elastic material behaviour of textile reinforced concrete (TRC) using an analytical approach. Damage and cracking of the composite are considered as well. The heterogeneous structure of TRC is modelled on the mesoscopic level and the overall material behaviour on the macroscopic level is obtained by means of homogenisation. The analytical approach is based on the micro-mechanical solution for a single inclusion according to Eshelby. In extension of this solution for multi-directionally reinforced concrete an effective field approximation is used. This approach allows the consideration of the interactions between the differently orientated rovings and also between rovings and micro-cracks in an average sense. The micro-cracks are included in the mechanical model by using a micro-crack density parameter. For the mechanical modelling of the bond behaviour between roving and matrix after the initiation of macro-cracking a slip-based bond model with a multiple linear shear stress–slip relation is introduced.

Keywords: Textile reinforced concrete; Homogenisation; Effective field approximation; Micro-crack density parameter; Slip-based bond model

1. Introduction

Textile reinforced concrete (TRC) is a composite of a so-called fine grained concrete matrix and a textile reinforcement which is used in the field of civil engineering for the fabrication of new structural elements and the strengthening of existing constructions [2]. The textile reinforcement consists of rovings. A roving is a bundle of a huge number of continuous filaments. The failure mechanisms of TRC are very complex. Most important are matrix-cracking, debonding of the roving from the matrix and breaking of the filaments and rovings [9]. The macroscopic material behaviour can be classified into a linear elastic part for low loadings, micro-cracking and macro-cracking. The final failure of the composite occurs due to the breaking of the rovings.

For the numerical simulation of textile reinforced structures the concept of representative volume elements (RVE) is meaningful. Therefore, the heterogeneous structure of TRC is analysed on the mesoscopic level and the macroscopic material behaviour, characterized by the overall elasticity tensor $\bar{\mathbf{C}}$, is obtained by the process of homogenisation. In addition to the application of numerical methods the RVE can be treated analytically under consideration of appropriate mechanical assumptions. In this paper analytical approaches are discussed. One advantage of the analytical approach is the determination of the overall macroscopic response of a RVE due to the macroscopic loading without high numerical costs.

The overall elasticity tensor $\bar{\mathbf{C}}$ is defined by the average stress $\bar{\sigma}$ and the average strain $\bar{\varepsilon}$. In the following the macroscopic strain ε^0 is prescribed. This strain must be equal to the average strain. Therewith the macroscopic constitutive relation can be written as

$$\bar{\sigma} = \bar{\mathbf{C}} : \bar{\varepsilon} = \bar{\mathbf{C}} : \varepsilon^0. \quad (1)$$

The aim is the determination of $\bar{\mathbf{C}}$ which shall be demonstrated in the following. The determined overall elasticity tensor $\bar{\mathbf{C}}$ can be used for integrated multi-scale analyses which do not demand for a constitutive relation of the nonlinear elastic or inelastic material behaviour on the macroscopic level. For a given macroscopic state of strain the mesoscopic mechanical model of the heterogeneous RVE is solved directly.

* Corresponding author.
E-mail address: mike.richter@tu-dresden.de (M. Richter).

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2. Linear elastic behaviour

For a low macroscopic loading the material behaviour is linear elastic. An RVE containing a homogeneous elastic matrix and n elastic inclusions Ω_α is assumed. On the boundary ∂V of the RVE the linear displacement field $\mathbf{u}^0 = \mathbf{x} \cdot \varepsilon^0$, associated with the constant symmetric macroscopic strain ε^0, is prescribed. The definition of the overall elasticity tensor $\bar{\mathbf{C}}$ in Eq. (1) leads with the volume averaging of the stresses and strains over the RVE:

$$\bar{\sigma} = f_\mathrm{m}\bar{\sigma}^\mathrm{m} + \sum_{\alpha=1}^{n} f_\alpha \bar{\sigma}^\alpha \quad \text{and} \quad \bar{\varepsilon} = \varepsilon^0 = f_\mathrm{m}\bar{\varepsilon}^\mathrm{m} + \sum_{\alpha=1}^{n} f_\alpha \bar{\varepsilon}^\alpha, \tag{2}$$

and the local material behaviour of the inclusions and the matrix to the relation [7]:

$$(\bar{\mathbf{C}} - \mathbf{C}) : \varepsilon^0 = \sum_{\alpha=1}^{n} f_\alpha (\mathbf{C}^\alpha - \mathbf{C}) : \bar{\varepsilon}^\alpha, \tag{3}$$

which can be used for the determination of $\bar{\mathbf{C}}$. This result is exact in the context of the given mechanical model (linear elastic matrix and inclusions, homogeneous boundary conditions). $\mathbf{C}$ is the elasticity tensor of the matrix, $\mathbf{C}^\alpha$ the elasticity tensor of the inclusion α, $\bar{\varepsilon}^\alpha$ the average strain in the inclusions and f_α the volume fraction of the inclusions. The problem is the determination of $\bar{\varepsilon}^\alpha$, whose analytical determination is based on the micro-mechanical solution for a single inclusion Ω embedded in an unconstrained elastic matrix with the far field strain ε^0 as derived by Eshelby [3]:

$$\bar{\varepsilon}^\Omega = (\mathbf{1} - \mathbf{S} : (\mathbf{1} - \mathbf{D} : \mathbf{C}^\Omega))^{-1} : \varepsilon^0, \tag{4}$$

with the Eshelby tensor $\mathbf{S}$ and the compliance tensor of the matrix $\mathbf{D} = \mathbf{C}^{-1}$. $\mathbf{1}$ is the identity tensor of fourth order. Neglecting any interactions between the inclusions (here the rovings), Eq. (3) leads with Eq. (4) to the overall elasticity tensor $\bar{\mathbf{C}}$:

$$\bar{\mathbf{C}} = \mathbf{C} + \sum_{\alpha=1}^{n} f_\alpha ((\mathbf{C}^\alpha - \mathbf{C})^{-1} + \mathbf{S}^\alpha : \mathbf{D})^{-1}. \tag{5}$$

This solution is well known and is called the solution for dilute distributions or simple the dilute solution.

As a first approximation Eq. (5) leads to appropriate results, but for larger volume fractions f_α of the inclusions the error in $\bar{\mathbf{C}}$ increases. For large volume fractions of the inclusions the assumption of the Eshelby-solution (the inclusion is situated in an unconstrained matrix with the far field strain ε^0) is not appropriate. The approach for the average strain $\bar{\varepsilon}^\Omega$ (see Eq. (4)) can be improved by the assumption that the inclusions are surrounded by a matrix with the average matrix strain $\bar{\varepsilon}^\mathrm{m}$. Now $\bar{\varepsilon}^\Omega$ is given as

$$\bar{\varepsilon}^\Omega = (\mathbf{1} - \mathbf{S} : (\mathbf{1} - \mathbf{D} : \mathbf{C}^\Omega))^{-1} : \bar{\varepsilon}^\mathrm{m}. \tag{6}$$

This approach is known as the effective field approximation (EFA) [6]. The average matrix strain $\bar{\varepsilon}^\mathrm{m}$ is unknown. If there is only one type of inclusions Ω, the average matrix strain can be substituted by the average strain in the inclusion $\bar{\varepsilon}^\Omega$ and the prescribed macroscopic strain ε^0. This problem can be solved easily and leads to an improved solution for the overall elasticity tensor for an RVE consisting of a matrix and one type of inclusions, see e.g. [7].

In the case of a multi-directional reinforcement we have a matrix reinforced with different rovings (e.g. different cross sections) of different orientations. The solution of this problem is more complicated, but a closed form analytical solution for $\bar{\mathbf{C}}$ can be found as well [9]. Each different inclusion Ω_α (here the different rovings) is assumed to be in a matrix with the still unknown average matrix strain $\bar{\varepsilon}^\mathrm{m}$ which can be substituted by means of the volume average:

$$\varepsilon^0 = f_\mathrm{m}\bar{\varepsilon}^\mathrm{m} + \sum_{\alpha=1}^{n} f_\alpha \bar{\varepsilon}^\alpha. \tag{7}$$

The volume fraction of the matrix f_m is known by

$$f_\mathrm{m} + \sum_{\alpha=1}^{n} f_\alpha = 1. \tag{8}$$

With Eqs. (7) and (8) the formulation of Eq. (6) for each individual inclusion Ω_α $(\alpha = 1, \ldots, n)$ leads to a system of n equations. As shown in [9] the average strains $\bar{\varepsilon}^\alpha$ in the n individual inclusions can be formulated depending on the prescribed macroscopic strain ε^0:

$$\bar{\varepsilon}^\alpha = \left\{ \mathbf{K}^\alpha + \sum_{\beta=1,\beta\neq\alpha}^{n} f_\beta (\mathbf{K}^\beta - f_\beta \mathbf{1})^{-1} : (\mathbf{K}^\alpha - f_\alpha \mathbf{1}) \right\}^{-1} : \varepsilon^0, \tag{9}$$

with the fourth order tensors $\mathbf{K}^\alpha$:

$$\mathbf{K}^\alpha = (f_\mathrm{m} + f_\alpha)\mathbf{1} - f_\mathrm{m}\mathbf{S}^\alpha : (\mathbf{1} - \mathbf{D} : \mathbf{C}^\alpha). \tag{10}$$

With Eq. (9), Eq. (3) leads to an equation for the direct computation of the overall elasticity tensor for a composite of a matrix and n different inclusions Ω_α:

$$\bar{\mathbf{C}} = \mathbf{C} + \sum_{\alpha=1}^{n} f_\alpha (\mathbf{C}^\alpha - \mathbf{C}) : \times \left\{ \mathbf{K}^\alpha + \sum_{\beta=1,\beta\neq\alpha}^{n} f_\beta (\mathbf{K}^\beta - f_\beta \mathbf{1})^{-1} : (\mathbf{K}^\alpha - f_\alpha \mathbf{1}) \right\}^{-1}. \tag{11}$$

This solution leads to better results than Eq. (5), because it considers the interactions between the different inclusions (e.g. the differently orientated rovings) in an average sense.

As a short example the results of a homogenised bidirectional reinforced concrete matrix shall be given. The concrete matrix is assumed to be isotropic ($E_\mathrm{m} = 30,000\,\mathrm{N/mm^2}$, $\nu_\mathrm{m} = 0.2$). The embedded roving as a loose bundle of a huge number of continuous filaments is idealised as a cylinder with only a longitudinal stiffness ($E_\mathrm{r} = 76,000\,\mathrm{N/mm^2}$). It acts transverse to its axis like a hole. Fig. 1 shows the overall Young's modulus $\bar{E}_3$