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Numerical simulation of indentation with size effect

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Abstract

Indentation size effect (ISE), whereby the strength of materials is observed to increase significantly with decreasing indentation depths has been reported in several experimental studies. In the present study, a series of nanoindentation experiments with maximum indentation depths varying from 400 to 3400 nm are carefully designed and implemented to study the indentation size effect on copper and Al7075. 3-D finite element analyses incorporating the conventional mechanism-based strain gradient (CMSG) plasticity theory which requires only C^0 solid elements are performed to simulate the indentation size effect. In order to circumvent the ambiguity associated with various methods to evaluate indentation hardness, the indentation size effect is investigated at the more fundamental level by considering changes to the load–displacement curve. The finite element results incorporating the CMSG plasticity theory are found to be in good agreement with experimental results at all levels of indentation depth conducted in the present study.

Keywords: Constitutive relation; C^0 finite elements; Indentation size effect; Strain gradient plasticity; Indentation test

1. Introduction

The necessity to perform material characterization on a small volume of material on MEMS and NEMS, the significant improvement of the instrumented indentation technique and the strength gain due to size effect have inspired numerous researchers to further advance the knowledge on indentation size effect (ISE). Several experiments have demonstrated the strong size effect when the characteristic length associated with non-uniform plastic deformation and the material length scale are of the same order of magnitude at the micron level. A substantial increase in torsional resistance but insignificantly in tensile strength of the copper wire is reported by Fleck et al. [1] when the wire diameters are reduced from 170 to 12 μm. Stolken and Evans [2] presented a similar phenomenon they observed during their micro-bend experiments of thin metallic wires and foils. Indentation size effect, whereby the strength of materials is observed to increase significantly with decreasing indentation depths has been reported in several experimental studies [3–5]. The trends of increasing indentation hardness with decreasing indentation depths observed in these studies are believed to be a manifestation of ISE. The phenomena and mechanisms behind the ISE is still a point of contention.

Various experimental and numerical studies to investigate the ISE have so far focused on studying hardness trends in instrumented indentation. However, in addition to changes in material strength, the variation of the measured hardness could be attributed to the different definitions of hardness which arise from diverse methods to evaluate the indentation contact area [6,7]. McElhaney et al. [5] and Lim and Chaudhri [8] demonstrated that the different methodologies adopted in the evaluation of the indentation contact area have a huge impact on the measured hardness values. In the studies on work-hardened oxygen-free copper conducted by Lim and Chaudhri [8], two different methods of evaluating the contact area resulted in conflicting trends of the variation of hardness with indentation depths.

In an instrumented indentation experiment, the applied load and the displacement of the indenter are the only two outputs available from the indentation apparatus. While certain indentation apparatus possess the capability to scan the surface before and after the indentation, the interpretation of the scanned image of the residual indentation imprint for hardness evaluation is still rather subjective. Therefore, the load–displacement response is the most fundamental quantity that is directly obtainable from the indentation apparatus. Based on the widely adopted Oliver and Pharr [9] method, the hardness is essentially a derived

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quantity which is calculated based on the maximum point of the loading curve and the slope of the unloading curve. The presence of any indentation size effect will directly affect the characteristics of the load–displacement curve and the variation of hardness can be thought of as a consequence due to changes in the shape of the load–displacement curve. Nix and Gao [10] have shown that the indentation size effect in single and polycrystalline copper and single crystal silver can be modeled through a simple characteristic relation of depth dependence of the hardness incorporating Taylor dislocation model. This was confirmed and further elaborated to cover the surface-nanocrystallized aluminum alloy by Wei et al. [11]. Both the hardness–displacement relation and the load–displacement curves illustrating the indentation size effect of these crystalline materials were presented in the latter.

While finite element analysis incorporating classical plasticity theory has been successfully adopted to simulate indentation experiments in which the indentation depth is in excess of several microns, the indentation size effect cannot be accounted for by the classical plasticity theory due to the absence of internal material length scale in the constitutive model. When the non-uniform plastic deformation wavelength and the material length scales are of the same order at the micron level, the states of stress are observed to be a function of both strain and strain gradient. Finite element simulations based on several alternative constitutive models incorporating the effects of strain gradient have been successfully carried out by Begley and Hutchinson [12], Huang et al. [13], Huang and Chen [14], Qiu et al. [15] and Chen et al. [16] to simulate the indentation size effect. These studies shared three similarities: (1) higher-order C^1 finite elements are required; (2) comparison between simulation and experimental results are made on the basis of hardness trends; (3) the Berkovich indenters used in the actual experiments are idealized as conical indenters in the axisymmetric finite element analysis. While C^1 finite elements are required in the implementation of the constitutive models considered by these researchers, these higher-order elements are often not available in commercial finite element packages. The formulation and implementation of user-defined C^1 finite elements is tedious and involved. The primary purpose of replacing the actual Berkovich indenter with an "equivalent" conical indenter in their numerical models is to simplify the problem from 3-D to 2-D (axisymmetric), reducing significantly the computational time, effort and resources. However, Swaddiwudhipong et al. [17] demonstrated that significant discrepancies exist between the load–displacement response of Berkovich and conical indenters, especially for shallow indentation where the effect of strain gradient is significant.

In the present study, nanoindentation experiments are carefully designed and implemented to study the indentation size effect on copper and Al7075. For each material, a series of nanoindentation experiments are carried out to the maximum indentation loads of 300, 200, 100, 50 and 10 mN. 3-D finite element analyses incorporating the conventional mechanism-based strain gradient (CMSG) plasticity theory are performed to simulate the indentation size effect. The implementation of the CMSG plasticity theory requires only C^0 solid elements which are widely available in commercial finite element packages. It is well known that the formulation and the performance of C^0 finite elements are significantly more convenient and efficient for implementation as compared to C^1 finite elements. In order to circumvent the ambiguity associated with various methods to evaluate indentation hardness, the indentation size effect is investigated at the more fundamental level by considering changes to the load–displacement curve.

2. Brief review on CMSG plasticity

The constitutive relations based on conventional continuum mechanics taken into account Taylor dislocation model [18] through the effective strain rate leading to the formulation of the C^0 CMSG solid finite element [19] will be briefly described. The contents included in this section are influenced greatly by the works of Fleck and Hutchinson [20], Gao et al. [21] and Huang et al. [22,23]. More details on the theoretical development and the validity of the models can be found in the above references [20–23].

The flow stress of Taylor dislocation model can be expressed as [21–23]:

$$\sigma_f = \sigma_{ref}\sqrt{f^2(\varepsilon^p) + l\eta^p} \quad (1)$$

$$l = \bar{r}b\left(\frac{M\alpha\mu}{\sigma_{ref}}\right)^2 \quad (2)$$

$$\eta^p = \sqrt{\frac{\eta^p_{ijk}\eta^p_{ijk}}{2}} \quad (3)$$

$$\eta^p_{ijk} = \varepsilon^p_{ik,j} + \varepsilon^p_{jk,i} - \varepsilon^p_{ij,k} \quad (4)$$

$$\varepsilon^p_{ij} = \int \dot{\varepsilon}^p_{ij}\,dt \quad (5)$$

where σ_{ref} is the reference stress, $f(\varepsilon^p)$ represents the stress and plastic strain relation in uni-axial tension, l the material length scale, η^p the effective plastic stain gradient, $\bar{r}$ the Nye [23–25] factor, M the Taylor factor, b the Burgers vector, μ the shear modulus and α is an empirical constant, the value of which reported earlier to vary from 0.2 to 1.1 [13,22,26] depending on the material structures, ε^p_{ij} is the plastic strain tensor and the subscript comma (,) implies differentiation with respect to the coordinate indices which follow. Bishop and Hill [27] showed that $M = 3.06$ for face-centered-cubic materials and for $\bar{r} = 1.90$ [23,28]:

$$l = 18b\left(\frac{\alpha\mu}{\sigma_{ref}}\right)^2 \quad (6)$$

The flow stress in Eq. (1) represents an average of dislocation activities and hence is only applicable for a problem with the length scale significantly larger than the average dislocation spacing. For a typical dislocation density of 10^{15} m^{-2}, Eq. (1) is valid for problems with length scale above 100 nm. The approach is thus applicable to most pure metals and metallic alloys where material length scale is in the micron range [3,22,29,30]. As pointed out by Nix and Gao [10] and further elaborated by Wei et al. [11], the characteristic length, h^*, and the characteristic material length scale, l, are closely related.