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Thermal shock resistance of functionally gradient solid cylinders

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Abstract

In this paper, an analysis of the transient thermo-mechanical behavior of a solid cylinder of functionally gradient material (FGM) under the convective boundary condition is presented theoretically. The analytical formula of the unsteady temperature distribution is derived by using the separation-of-variables method and hence the maximum thermal stress attained at the surface of the FGM solid cylinder as well as its time of occurrence can be calculated. Based on a local tensile stress criterion, the expression of critical temperature change ΔT_c leading to the local tensile strength at the surface, which is designated as the thermal shock resistance parameter for FGM solid cylinder, is obtained. The effects of the radial distributions of thermo-physical properties on the thermal shock resistance of the FGM solid cylinder are investigated via numerical calculations in contrast to homogeneous solid cylinder, from which some suggestions on design of FGM solid cylinders with high thermal shock resistance are put forward.

Keywords: Functionally gradient materials; Thermal shock resistance; Critical temperature change; Solid cylinder; Unsteady thermal stresses

1. Introduction

Functionally gradient materials (FGM) are new advanced composite materials intentionally designed so that they possess desirable properties for specific applications, especially for performance under thermal environment. In the theory of elasticity, FGMs are mostly treated as non-homogeneous materials with material constants varying continuously along one spatial direction. In the past two decades, a number of investigations have dealt with thermoelastic problems for the basic structural components of FGMs. Noda and co-workers [1–6] analyzed the one-dimensional steady state thermal stress problems of a FGM plate and other shapes, and proposed an analytical method of one-dimensional unsteady thermal stresses in a FGM plate by using the perturbation method. Araki et al. [7] and Sugano et al. [8,9] derived the analytical solution of the temperature distribution in a multilayered material under pulse or stepwise heating. Tanigawa et al. [10–12] used an iteration technique to analyze the one-dimensional transient thermal stresses of a FGM plate and a FGM cylinder using a laminated composite model, and discussed the temperature dependence of material properties, they [13] also analyzed the transient thermal stress of a non-homogeneous plate taking into account the relative heat transfer at boundary surfaces. Awaji et al. [14–16] also proposed techniques for analyzing one-dimensional steady state and transient temperature distribution in a FGM plate and a hollow FGM cylinder.

By introducing the theory of laminated composites, Ootao and Tanigawa [17] treated the three-dimensional transient thermal stresses of functionally gradient rectangular plates due to partial heat supply, and analyzed the piezothermoelastic problem of a functionally gradient rectangular plate bonded to a piezoelectric plate [18]. Kim and Noda [19,20] researched the two-dimensional unsteady thermoelastic problems of functionally gradient infinite hollow cylinders by using a Green's function approach. Jabbari et al. [21] derived analytical solutions for one-dimensional steady-state thermoelastic problems of functionally gradient circular hollow cylinders in the case of material models expressed as power functions of r, and treated the two-dimensional thermoelastic problems of the functionally gradient cylinder by using the Fourier transform [22]. Liew et al. [23] obtained analytical solutions of a functionally gradient circular hollow cylinder by a novel limiting process that employs the solutions of homogeneous circular hollow cylinders.

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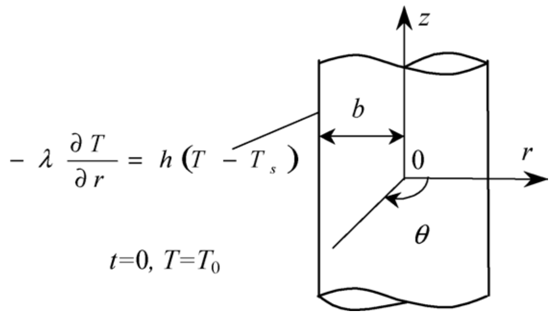

Fig. 1. Functionally gradient cylinder.

Wang et al. obtained the exact expression of dynamic thermal stress in a transversely isotropic sphere and consequently studied the thermal stress-focusing effect [24]. Their further investigations on the magnetothermodynamic stress and perturbation of magnetic vector in a conducting orthotropic thermoelastic cylinder subject to thermal shock [25], and the thermo-electro-elastic transient response in piezoelectric hollow structures subjected to arbitrary thermal shock, sudden mechanical load and electric excitation [26,27] provided valuable references to solve some transient coupled problems for non-homogeneous structures.

To our knowledge, however, little work has been focused on the solution for the unsteady temperature field and unsteady thermal stress field of FGM solid cylinders under the convective boundary condition, and a strength-based fracture criterion for the thermal shock resistance evaluation of FGM solid cylinders has not been previously obtained probably because there is few application of this kind of FGMs currently.

The present work is undertaken to propose a thermal shock resistance parameter for FGM solid cylinders. A FGM solid cylinder with composition and properties varying radially (Poisson's ratio is constant) is studied, with its surface suddenly exposed to a convective medium of different temperature. The analytical formula of the unsteady temperature distribution is obtained by using the separation-of-variables method and hence the maximum thermal stress attained at the surface of the FGM solid cylinder as well as its time of occurrence can be calculated. Based on a local tensile stress criterion, the expression of critical temperature change ΔT_c leading to the local tensile strength at the surface as the thermal shock resistance parameter for FGM solid cylinder is obtained. The effects of the distributions for thermo-physical properties on the thermal shock resistance of the FGM solid cylinders are discussed via numerical calculations in contrast to homogeneous solid cylinder.

2. Analytical methods

2.1. Unsteady temperature field

As shown in Fig. 1, consider a long FGM solid cylinder with its radius being b. A cylindrical coordinate system (r, θ, z) is established for reference, with the z-axis lying on the axis of the cylinder. As we seek the unsteady thermal stress solutions of the FGM cylinder in the plane strain condition, the temperature is independent of z. It is assumed that initially it is at a uniform temperature T_0, and at time $t=0$ the surface of the cylinder (at $r=b$) is suddenly exposed to a convective medium of temperature T_s. And the coefficient of heat transfer h is assumed to be a constant.

The thermo-mechanical properties of the FGM solid cylinder are varied in the radial direction (Poisson's ratio is assumed to be constant) continuously. Hence the temperature field and the thermal stress field are also functions of radial coordinate r.

The one-dimensional unsteady heat conduction equation of non-homogeneous solid cylinders, the initial and boundary conditions are given as follows:

$$\rho(r)c(r)\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\lambda(r)\frac{\partial T}{\partial r}\right] \tag{1}$$

$$T(r,0) = T_0 \tag{2}$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \tag{3a}$$

$$-\lambda\left.\frac{\partial T}{\partial r}\right|_{r=b} = h(T - T_s) \tag{3b}$$

where T is the temperature, t is the time, r is the coordinate variable in the radial direction, and $c(r)$, $\rho(r)$ and $\lambda(r)$ are specific heat, specific gravity and thermal conductivity, respectively. For the purpose of simplicity, the following dimensionless variables are