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Consequences of an asymmetrical confinement in the transfer phenomena for a cylinder at low Reynolds numbers

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ABSTRACT

In most of the systems where heat or mass transfer occur geometrical symmetries are often present. In this paper, we wish to know if the global flux transferred to cylindrical particles in motion (sedimentation, etc.) could be influenced when introducing some geometrical disturbance breaking the general symmetry of the system. To answer this question, we numerically investigate the simple configuration of a single cylindrical particle moving at constant speed between two parallel walls when the particle is off the symmetry plane and when the thermal boundary conditions are of the Dirichlet or Neumann type. When the geometrical disturbance of the system increases, the results show that the backflow that appears in such confined situations yields a non-intuitive evolution of the transfer in convective regimes. In this case a minimum of the flux appears off the symmetry plane. However, in purely diffusive regimes, we find a monotonical evolution of the transfer.

1. Introduction

The control of heat or mass transfer on a cylindrical particle is of major interest in numerous problems such as heat exchangers, hot-wire anemometry (Lange et al., 1999; Shi et al., 2002) in the case of thermal transfer, and in the case of mass transfer for problems such as filtration of microscopic particles on membranes composed of cylindrical fibers (Friedlander, 1977) and electrochemical processes (Vogtländer and Bakker, 1963). Other problems relate to heat or mass transfer to cylindrical particles in sedimentation in a confined medium. In all these problems, we often face hydrodynamic and thermal interactions between a single particle and others or between a single particle and some walls. Such confined configurations rarely have specific geometrical symmetries. The knowledge of the consequences of such asymmetries in the transfer on a cylinder is therefore useful for possible optimization of these transfers. Indeed, the introduction of some geometrical noise (position or diameter for example) in an array of ordered cylinders could possibly lead to a better transfer compared to the symmetrical configuration.

In order to understand the physics behind the transfers involved in such geometrically disturbed situations, we focus in this article on the basic system composed of a cylinder moving asymmetrically and parallel to two parallel walls for Dirichlet and Neumann boundary conditions. For this study, we need to recall some basic results concerning the transfer on a cylinder in an unlimited fluid and to give new results about symmetrically confined cylinders.

Let us recall that the convective heat transfer on a circular cylinder of radius a translating with velocity U_0 in an incompressible Newtonian fluid is generally controlled by the dimensionless Prandtl number $Pr = \nu/D_{th}$ (ν is the kinematic viscosity and D_{th} is the thermal diffusion coefficient), Reynolds number $Re = U_0 2a/\nu$ and by geometrical factors due to the confinement. However for non-inertial regimes, the heat transfer is controlled solely by the Péclet number ($Pe = Pr\,Re$) that depends only on some geometrical factors. In that particular confined case, the heat transfer at very low Reynolds numbers corresponding to "dynamic Stokes type regimes" depends only on the geometrical parameters of the problem as will be discussed later. In this study, we are interested only in non-inertial dynamic regimes without any restriction regarding the value of the Péclet number which corresponds to some specific industrial fluids with high viscosity and low thermal conductivity such as glycerol, ethylene glycol or silicone oils (for instance Rhodorsil^{TM} silicone oil 47V1000, $Pr = 10^4$. Note that the main part of the other studies dedicated to this subject concerns rather high Reynolds numbers regimes.

For the low Reynolds numbers, the heat transfer on a cylinder at temperature T_0 placed in an infinite medium at temperature T_∞ and moving with constant velocity U_0 was obtained analytically by Cole and Roshko (1954). They solved the energy equation in an "Oseen type" approximation (Oseen, 1910). The Nusselt number in their

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study is denoted by

$$Nu = \frac{\int_0^{2\pi} (\partial T/\partial r)_{r=a} a \, d\theta}{\pi(T_0 - T_\infty)} = \frac{2}{\ln(8/Pe) - \gamma}, \tag{1}$$

with $\gamma = 0.5772\ldots$ the Euler's constant, which is valid for low Reynolds and Péclet numbers. In a manner similar to the hydrodynamic drag force undergone by the cylinder which depends on the Reynolds number through a logarithmic term, the heat flux always depends on the Péclet number. The flux per unit of length vanishes when $Pe \to 0$ because this seems to increase less rapidly than the cylinder length. After this pioneering work, other authors used again the Oseen approximation and gave longer developments. For instance Illingsworth (1963), starting from the same "Oseen type" approximation for the energy equation as Cole and Roshko (1954), gave the second term of formula (1). We have developed his expression for $Pe \ll 1$ and $Re \ll 1$ using the "Mathematica" computer algebra software:

$$Nu = \frac{2}{\ln(8/Pe) - \gamma} - \left(\frac{Pe}{8}\right)^2 \left\{8 + \frac{2}{[\ln(8/Pe) - \gamma]^2}\right\}. \tag{2}$$

Wood (1968) added two more terms to expression (1) of Cole and Roshko (1954) using both the Oseen approximation for the momentum equations and an "Oseen type" approximation for the energy equation. Once again we give an expression of his formula obtained with the "Mathematica" computer algebra software and valid for the low values of the Reynolds and Péclet numbers:

$$Nu = \frac{2}{\ln(8/Pe) - \gamma} \left\{1 - \frac{\lambda(Pr) + \mu(Pr)/[\ln(8/Re) - \gamma + 1/2]}{[\ln(8/Pe) - \gamma][\ln(8/Re) - \gamma + 1/2]}\right\}, \tag{3}$$

where $\lambda(Pr)$ and $\mu(Pr)$ are integral functions to be computed numerically. The author emphasized that his expression was accurate on the understanding that both Re and Pe are small and that for fluids with large Prandtl number, the maximum Reynolds number for which the approximation holds is correspondingly reduced. Wood (1968) gave values for $\lambda(Pr = 0.72) = 1.38$ and $\mu(Pr = 0.72) = 0.40$ in air for $Re < 0.3$. Thanks to the velocity fields obtained by Proudman and Pearson (1957) or Kaplun (1957) using matched asymptotic expansions for the velocity field and an "Oseen type" approximation for the energy equation, Hieber and Gebhart (1968) gave a solution for the low Reynolds numbers limit:

$$Nu = \frac{2}{\ln(8/Pe) - \gamma} \left\{1 - \frac{\delta(Pr)}{[\ln(8/Pe) - \gamma]^2}\right\}, \tag{4}$$

where $\delta(Pr)$ is again a function to be derived numerically. They gave the values $\delta(Pr = 0.72) = 1.38$, $\delta(Pr = 1) = 1.63$ and $\delta(Pr = 6.82) = 3.42$. When the authors compared this expression to others and experimental data, they concluded that the applicability of this formula is $Re < 0.04$. With the same matched asymptotic expansion as Kassoy (1967), Nakai and Okazaki (1975) obtained the following expression:

$$Nu = \frac{2}{2/3 + \ln(8/3) - \ln Pe}. \tag{5}$$

The range of application for this formula is $Re\,Pr < 10^{-1}$ and comparisons with experimental data and other expressions seem to be satisfactory. A numerical study of this problem using the finite-volume method was given by Lange et al. (1998) for $10^{-4} \leqslant Re \leqslant 200$. Their numerical results correctly matched the experimental data of Collis and Williams (1959) for air ($Pr = 0.72$). These latter authors proposed an expression to fit their data for $0.02 < Re < 0.5$ that resembled the relation of Cole and Roshko (1954):

$$Nu = \left[\frac{T_m}{T_\infty}\right]^{0.17} \frac{1}{1.18 - 1.10 \log_{10} Re}, \tag{6}$$

with $T_m = (T_0 + T_\infty)/2$ being the mean arithmetic temperature. Notice that for $Pe \gg 1$ and $Re \ll 1$, the Nusselt number can be written as $Nu \propto A\,Pe^{1/3}$ (see Lévêque, 1928). In this regime, we cite the theoretical expression of Friedlander (1957) obtained from the boundary layer theory and the stream function determined by Tomotika and Aoi (1951) for heat and mass transfer at high Péclet numbers:

$$Nu = \frac{0.821\,Pe^{1/3}}{(2 - \ln Re)^{1/3}} \quad \text{for } Re < 10^{-3}. \tag{7}$$

In the article of Kurdyumov and Fernández (1998) using the boundary layer theory applied to the energy equation, the following expression was proposed:

$$Nu = \frac{0.597\,Pe^{1/3}}{[\ln(1/Re)]^{1/3}} + 0.0917 \quad \text{for } Re < 10^{-2} \text{ and } Pr \gg 1. \tag{8}$$

Because of the abundance of experimental articles dealing with this subject, we prefer to cite the work of Morgan (1975) which brings together the most significant data on this topic. It is clear while reading this paper that the Nusselt number is a function of the Reynolds and Prandtl numbers but with a wide scatter concerning their exponents. This is due partly to the change in the hydrodynamic regime and, in the case of the very low Reynolds numbers to the influence of the confinement and to size effects. In fact as all the experiments were made in confined situations, it is important to notice that all the formulae for $Pe \ll 1$ given above are not valid anymore because at low Péclet numbers the diffusive boundary layer is very large so that the confinement of the cylinder introduces an important correction due to its cut-off as will be justified in the present study concerned with cylinders in confined situations.

Concerning a cylinder in unlimited medium, we can cite a few studies dealing with the mass transfer. Langmuir (1942) looked into the problem of aerosol filtration. Using the Oseen approximation he supposed that the mass transfer on the cylinder occurred for $\pi/6 < \theta < 5\pi/6$ (θ is the angle from the upstream stagnation point). He proposed the following formula for the Sherwood number:

$$Sh = \frac{\int_0^{2\pi} (\partial C/\partial r)_{r=a} a \, d\theta}{\pi(C_0 - C_\infty)} = \frac{0.335 Pe^{0.4}}{[2 - \ln Re]^{0.4}}, \tag{9}$$

with C being the mass concentration. This expression is valid for $0.03 \leqslant Re \leqslant 0.4$ and $1000 \leqslant Sc \leqslant 28000$ with $Sc = \nu/D_m$ the Schmidt number (D_m is the molecular diffusion coefficient) and $Pe = Re\,Sc$. Let us recall that formula (7) of Friedlander (1957) is also valid for mass transfer for $Re < 10^{-3}$ and high Péclet number. As for the experimental studies, one can cite the work of Dobry and Finn (1956) who used an electrochemical method to measure the mass transfer on a tiny metal wire for Reynolds numbers between 0.02 and 8 and Schmidt numbers around 1000 or the work of Ambari et al. (1984) using the same experimental technique. These latter authors proposed for the Sherwood number the following expression:

$$Sh = 0.76 Sc^{1/3}\,Re^{0.28}, \tag{10}$$

for $10^{-3} < Re < 7$ and $Sc = 10^3$.

The governing equations controlling the mass transfer are equivalent to those controlling the heat transfer when the Rayleigh dissipation function is negligible for similar problems with the same type of boundary conditions. For most of the systems and for ordinary fluids, the amount of energy per unit volume dissipated by viscous friction $E_\mu \sim \mu U_0^2/a^2$ is insignificant compared to the amount of energy per unit volume convected in the flow $E_{\text{th}} \sim \rho C_P U_0 \Delta T$. μ is the dynamic viscosity of the fluid, ρ the fluid density, C_P the specific heat at constant pressure and $\Delta T = |T_0 - T_\infty|$. That means that $E_\mu/E_{\text{th}} = (2U_0^2/C_P\Delta T)/(U_0 2a/\nu) = Ec/Re \ll 1$ with $Ec = 2U_0^2/C_P\Delta T$ the Eckert number. In the following work we suppose that this relation is true and we will only deal with the heat transfer without any dissipation. Moreover we assume that the temperature variations are