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Quantum pump in quantum spin Hall edge states

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ABSTRACT

We present a theory for quantum pump in a quantum spin Hall bar with two quantum point contacts (QPCs). The pump currents can be generated by applying harmonically modulating gate voltages at QPCs. The phase difference between the gate voltages introduces an effective gauge field, which breaks the time-reversal symmetry and generates pump currents. The pump currents display very different pump frequency dependence for weak and strong *e–e* interaction. These unique properties are induced by the helical feature of the edge states, and therefore can be used to detect and control edge state transport.

1. Introduction

Topological insulators (TIs), strong spin–orbit coupling systems, have attracted intense attention due to their unique electronic structure and novel transport property. A TI possesses a gap in the bulk and a metallic surface and/or edge states at its boundary [1–5]. The edge states are helical and robust against the non-magnetic impurity scattering and local perturbation because the backscattering process is forbidden due to the helical feature of the edge states [6–11]. One of the key questions in this rapid growing field is how to detect and control the topological edge state. This is because the presence of edge states is determined by the global topology of the band structure of TIs. So far the edge states in a quantum spin Hall (QSH) bar have been detected by two- and multi-terminal conductances under a dc bias between the source and the drain. Recently, there have been a few proposals to control the edge state transport using the quantum point contact (QPC), i.e., the inter-edge coupling [12–16]. The gap, which blocks the edge channels, can be opened by the coupling between the edge states at opposite boundaries.

Quantum electron transport without dissipation is always one of the central issues of the condensed matter physics, e.g., superconductivity, the zero resistance induced by microwave radiation, the quantum Hall effect in two-dimensional electron gas (2DEG) and the recent discovery, the quantum spin Hall effect. Quantum pump, a captivating quantum coherent effect, is a focal point of mesoscopic physics, describing an electrical current generated by periodically varying parameters of the quantum system rather than using a bias [17–22]. Usually a dc current is associated to a dissipative flow of the electrons in response to an applied bias voltage. However, in systems of mesoscopic scale a dc current can be generated even at zero bias, for example, in a quantum pump. Quantum pump is closely associated with the symmetry breaking in a mesoscopic system, e.g., the breaking of the time reversal symmetry (TRS). Previous theoretical and experimental works focused on quantum pump in conventional semiconductor mesoscopic systems. Very recently, quantum pump in single and bilayer graphene shows interesting behaviors due to their unique band structure. The evanescent modes dominate graphene quantum pump at the Dirac point and the pump current depends sensitively on the edge shape of graphene nanoribbons, i.e., the edge states [23–27]. These edge states are protected by the pseudo-TRS. In contrast to graphene, the edge states in a quantum spin Hall system are protected by the true TRS and show a unique spin-momentum locking, i.e., the helical feature.

Recent studies in the TI field concentrated on dc transport, while ac transport property is relatively unexplored. In this Letter, we propose a quantum pump scheme to control and detect the edge states in TIs by tuning the phase and amplitude of harmonic gate voltages applied on two QPCs. The phase difference acts as an effective gauge field and breaks the TRS, consequently induces pump currents.

2. Theoretical model

First we consider a quantum spin Hall bar with two QPCs (see Fig. 1). The solid (dashed) lines represent spin-up electron edge states incident from the left (right) contact. The helical edge states propagating along the opposite edges can be coupled at the two QPCs. Since electrons in such quasi-one-dimensional helical edge states are strongly correlated, the *e–e* interaction should be included based on the Luttinger liquid (LL) theory, which was

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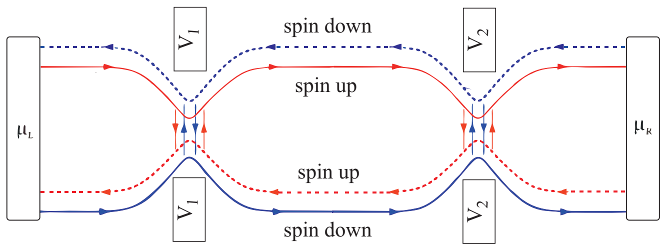

Fig. 1. (Color online) Schematic of the quantum spin Hall system with two QPCs. V_1 and V_2 are the side gate voltages applied on the QPCs, respectively. μ_R (μ_L) is the chemical potential at the right (left) lead.

neglected in the previous works [15,16]. The strength of e–e interaction is limited, i.e., $1/2 < K < 1$, so that the constrictions are far from being pinched off [12].

The low-energy Hamiltonian describing the helical edge electrons is

$$H_0 = -i\hbar v_F \sum_{\sigma=\uparrow,\downarrow} \int dx(\psi_{R\sigma}^{\dagger}\partial_x\psi_{R\sigma} - \psi_{L\sigma}^{\dagger}\partial_x\psi_{L\sigma}), \tag{1}$$

where v_F is the Fermi velocity, $\psi_{\gamma s}$ $(\gamma = -1(L), 1(R);\ s = -1(\downarrow), 1(\uparrow))$ annihilate the left (L) or right (R)-moving spin-down ($\downarrow$) or spin-up ($\uparrow$) electrons, respectively. Away from half-filling of the one-dimensional band, time-reversal invariance constrains the possible e–e interaction processes to dispersive ($\sim g_d$) and forward scattering ($\sim g_f$). In the of the Fermi points, the corresponding interaction is given by $H_d = g_d \int dx\left(\psi_{R\uparrow}^{\dagger}\psi_{R\uparrow}\psi_{L\downarrow}^{\dagger}\psi_{L\downarrow} + \psi_{L\uparrow}^{\dagger}\psi_{L\uparrow}\psi_{R\downarrow}^{\dagger}\psi_{R\downarrow}\right)$, and $H_{fw} = (g_f/2)\sum_{\gamma=R,L,\sigma=\uparrow,\downarrow}\int dx\psi_{\gamma\sigma}^{\dagger}\psi_{\gamma\sigma}\psi_{\gamma\sigma}^{\dagger}\psi_{\gamma\sigma}$. We can bosonize the Hamiltonian $H = H_0 + H_d + H_{fw}$, and obtain

$$H = \frac{v}{2}\sum_{i=1,2}\int dx\left(\frac{1}{K}\left(\partial_x\phi_i\right)^2 + K\left(\partial_x\theta_i\right)^2\right), \tag{2}$$

where ϕ_i and θ_i are the boson fields within the standard bosonization scheme. The indices 1 and 2 label the upper and lower edges, respectively, $\phi_1 = \Phi_{R\uparrow} + \Phi_{L\downarrow}$, $\phi_2 = \Phi_{L\uparrow} + \Phi_{R\downarrow}$, $\theta_1 = \Phi_{R\uparrow} - \Phi_{L\downarrow}$, $\theta_2 = \Phi_{L\uparrow} - \Phi_{R\downarrow}$. $\Phi_{\gamma\sigma}$ is a chiral boson field related to the electron annihilation operator $\psi_{\gamma\sigma}(t) = e^{2\sqrt{\pi}i\gamma\Phi_{\gamma\sigma}}/\sqrt{2\pi a}$, a is the short distance cutoff. $K = \sqrt{\left(2\pi v_F + g_f - g_d\right)/\left(2\pi v_F + g_f + g_d\right)}$ and $v = \sqrt{\left(v_F + g_f/2\pi\right)^2 - \left(g_d/2\pi\right)^2}$. The helical feature of the edge states, i.e., spin-momentum locking, only allows two types of tunneling terms at the QPCs. [28] One is the spin-preserving tunneling process

$$H_p = U_p\left(t\right)e^{i\omega_J t}\left(\psi_{L\uparrow}^{\dagger}\psi_{R\uparrow} + \psi_{L\downarrow}^{\dagger}\psi_{R\downarrow}\right) + H.\,c. \tag{3}$$

The other is the spin-flip tunneling process

$$H_f = U_f\left(t\right)(-\psi_{L\uparrow}^{\dagger}\psi_{L\downarrow} + \psi_{R\uparrow}^{\dagger}\psi_{R\downarrow}) + H.\,c. \tag{4}$$

We will assume that $U_p\left(t\right) = U^p\cos\left(\Omega_p t + \phi^p\right)$, $U_f\left(t\right) = U^f\cos\left(\Omega_f t + \phi^f\right)$, which means that all contacts vary harmonically in time with the same frequency Ω, but with different phases and strengths. We introduce time dependence into the fermionic operators $\psi_L \to \psi_L e^{-i\mu_L t}$ and $\psi_R \to \psi_R e^{-i\mu_R t}$. $\mu_L - \mu_R = \omega_J = e^* V/\hbar$, where e^* is the charge of electron.

Next we calculate the QSH edge current in the presence of quantum tunneling (see Eqs. (3) and (4)) at the QPCs. The harmonic oscillating gate voltages V_j $(j = 1, 2)$ are applied at the two QPCs located at x_j. The tunneling current can be obtained from the time evolution of the charge number operator of the right-moving electrons $N_{R\sigma} = a\psi_{R\sigma}^{\dagger}\psi_{R\sigma}$, $j_{R\sigma}\left(t\right) = e^*\, dN_{R\sigma}(t)/dt = e^*\left[N_{R\sigma}, H\right]/(i\hbar)$. The expectation value for the current at time t is given by $\left\langle j\left(t\right)\right\rangle = \left\langle 0|S^{\dagger}\left(t, -\infty\right)j\left(t\right)S\left(t, -\infty\right)|0\right\rangle$, where $|0\rangle$ denotes the initial state at $t \to -\infty$, and $S(t, -\infty) = Te^{-\frac{i}{\hbar}\int_{-\infty}^{t} dt' H_{tun}\left(t'\right)}$ is the time evolution operator, T is the time-order operator. The quantum pump current occurs at vanishing source-to-drain voltage, therefore we only consider the lowest order in the perturbation expansion. To the lowest order in the perturbation, the scattering matrix is given by $S(t, -\infty) = 1 - \frac{i}{\hbar}\int_{-\infty}^{t} dt' H_{tun}(t')$, therefore we have $\langle j_{R\sigma}(t)\rangle = -\frac{i}{\hbar}\int_{-\infty}^{t} dt'\left\langle 0|[j_{R\sigma}\left(t\right), H_{tun}\left(t'\right)]|0\right\rangle$. The charge current is $I_c = \left\langle j_{R\uparrow}(t) + j_{R\downarrow}(t) - \left(j_{L\uparrow}(t) + j_{L\downarrow}(t)\right)\right\rangle$ and the spin current $I_s = \left\langle j_{R\uparrow}(t) - j_{R\downarrow}(t) - \left(j_{L\uparrow}(t) - j_{L\downarrow}(t)\right)\right\rangle$. The spin preserving tunneling process only induces the charge current, but without the spin current, i.e., $I_c = -\frac{i}{\hbar}\sum_{\sigma}\int_{-\infty}^{t} dt'\left\langle 0|[j_{R\sigma}\left(t\right), H_p\left(t'\right)]|0\right\rangle$, $I_s = 0$. While the spin-flip tunneling process induces the spin current $I_s = -\frac{i}{\hbar}\sum_{\sigma}\sigma\int_{-\infty}^{t} dt'\left\langle 0|[j_{R\sigma}\left(t\right), H_f\left(t'\right)]|0\right\rangle$, and a vanishing charge current $I_c = 0$.

The total charge (spin) tunneling current is $I_{c/s} = I_{c/s,dc} + I_{c/s,ac}$. In experiments, the frequency Ω of external gate voltage can be very high, therefore, one can only measure the dc component of the tunneling current

$$I_{c/s} = I_{c/s,dc} = \sum_{j=1,2} I_{c/s,dc}^{j} + \sum_{i\neq j} I_{c/s,dc}^{ij}. \tag{5}$$

The first term is the charge tunneling current for a single QPC, and the second term is the interference term of the dc charge tunneling current.

$$I_{c,dc}^{j} = \frac{e^*\left(U_j^p\right)^2(a/v)^{2\Delta_K}}{\hbar^2\Gamma(2\Delta_K)}[\mathrm{sgn}(\omega_+)|\omega_+|^{2\Delta_K-1} + \mathrm{sgn}(\omega_-)|\omega_-|^{2\Delta_K-1}], \tag{6a}$$

$$I_{c,dc}^{12}(x_{12}) = \frac{e^*\sqrt{\pi}U_1^p U_2^p}{4\pi\hbar^2\Gamma(\Delta_K)}\left(\frac{a}{v}\right)^{2\Delta_K}\left(\frac{2|x_{12}|}{v}\right)^{\frac{1}{2}-\Delta_K} \times\left[\mathrm{sgn}(\omega_+)|\omega_+|^{\Delta_K-\frac{1}{2}}J_{\Delta_K-\frac{1}{2}}\left(\frac{|\omega_+ x_{12}|}{v}\right)\times\cos(2k_F x_{12} + \phi_{12}^p) + \mathrm{sgn}(\omega_-)|\omega_-|^{\Delta_K-\frac{1}{2}}\times J_{\Delta_K-\frac{1}{2}}\left(\frac{|\omega_- x_{12}|}{v}\right)\cos(2k_F x_{12} - \phi_{12}^p)\right], \tag{6b}$$

where $\mathrm{sgn}(\omega) = 1$ for $\omega > 0$, 0 for $\omega = 0$, and -1 for $\omega < 0$. J is the first kind of the Bessel function, Γ is the Gamma function, $\Delta_K = \left(K + 1/K\right)/2$, $\omega_+ = \omega_J + \Omega_p$, $\omega_- = \omega_J - \Omega_p$. $x_{12} = x_1 - x_2$ represents the spatial separation between the two QPCs and $\phi_{12} = \phi_1 - \phi_2$ represents the phase difference between the time-dependent voltages applied on the two QPCs. For the static case ($\Omega = 0$) of a QSH bar with a single QPC, the charge tunneling current is given by $I_{c,dc}^{j} = \frac{2e^*\left(U_j^p\right)^2}{\hbar^2\Gamma\left(2\Delta_K\right)}\left(\frac{a}{v}\right)^{2\Delta_K}\mathrm{sgn}(\omega_J)|\omega_J|^{2\Delta_K-1}$, which is consistent with the results in Ref. [12].

For the quantum pump case ($\omega_J = 0$), the dc charge current is