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Spin conductivity of the two-dimensional anisotropic frustrated Heisenberg model in the honeycomb lattice

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ABSTRACT

We use the SU(3) Schwinger's boson theory to study the spin transport properties of the two-dimensional anisotropic frustrated Heisenberg model in a honeycomb lattice at $T=0$. We have investigated the behavior of the spin conductivity for this model which presents a single-ion anisotropy and J_1 and J_2 exchange interactions. We study the spin transport in the Bose–Einstein condensation regime where we have that the t_z bosons are condensed and the following condition is valid: $\langle t_z\rangle = \langle t_z^{\dagger}\rangle = t$. Our results show a metallic spin transport for $\omega > 0$ and a superconductor spin transport in the limit of *DC* conductivity, $\omega \to 0$, where $\sigma(\omega)$ tends to infinity in this limit of ω.

1. Introduction

Frustrated spin systems have been object of intense research in the recent years [1] where the competitive interactions in quantum magnetic systems especially on a honeycomb lattice have much interest [1–18]. The main effects of frustrating interactions, in the neighborhood of a Neel state, are the increase of the coupling and the decrease of the spin-wave velocity. For values of J_2 that are sufficiently large, the dimensionless coupling constant will become larger than the critical value. Consequently, there must be a critical value of next nearest neighbor coupling, of strength J_{2c}, beyond which the long-range Néel order is destroyed. This theory then predicts that for one $J_2 \leq J_{2c}$, the system becomes a quantum paramagnet. It is also clear that if J_2 become large enough a new form of long-range order should be found, if $J_2 \gg J_{2c}$ a Néel like state but with wave vector $\vec{Q} = (\pi, 0)$ or $(0, \pi)$ is favored, instead of the usual $\vec{Q} = (\pi, \pi)$ ordered state. This Néel state is antiferromagnetic along the x-axis but ferromagnetic along the y-axis. This form of antiferromagnetism occurs, for instance, in the ion pnictide materials, which are also high-temperature superconductors [19].

Besides the mathematical beauty of the Kitaev model, recent studies were motivated by its possible relevance for degenerate orbital systems with strong spin–orbit coupling such as layered iridates Na_2IrO_3 and Li_2IrO_3. These applications require to consider the extension, namely the Kitaev–Heisenberg model on honeycomb lattice, which reveals in its phase diagram apart of a liquid phase also several ordered (e.g. stripe or zig-zag) phases. These phases are states of matter characterized by decay of the spin–spin correlations functions in the form of the power-law and a zero local magnetization, where such properties occur at the zero temperature, where the disorder is derived from quantum fluctuations [1].

Recently there is an intense research about the quantum Hall effect for spins and magnon spintronics [21–25]. In the study of these effects often only the sign differences between related quantities like magnetic fields and generated spin and charge currents are determined. The spin transport properties in the spin systems have been studied theoretically by Sentef et al. [26] who have analyzed the spin transport in the easy-axis Heisenberg antiferromagnetic model in two and three dimensions, at $T=0$. Damle and Sachdev [27] have treated the two-dimensional case using the non-linear sigma model in the gapped phase. Pires and Lima [28–30] treated the two-dimensional easy plane Heisenberg antiferromagnetic model. Lima and Pires [31] studied the spin transport in the two-dimensional anisotropic XY model using the SU(3) Schwinger boson theory in the absence of impurities, Lima [32] has studied the case of the Heisenberg antiferromagnetic model in two dimensions with Dzyaloshinskii–Moriya interaction. Chen et al. [33] analyzed the effect of spatial and spin anisotropy on spin conductivity for the $S=1/2$ Heisenberg model on a square lattice and more recently, Kubo et al. [34] studied the spin conductivity in two-dimensional non-collinear antiferromagnets at $T=0$ using spin wave theory and Lima et al. [35] have studied the spin transport in the site diluted two-dimensional anisotropic Heisenberg model in the easy plane, using the self-consistent harmonic approximation.

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The aim of this paper is to study the spin transport in the two-dimensional anisotropic frustrated Heisenberg model on a honeycomb lattice using the SU(3) Schwinger's boson approximation. Recently, the critical properties of this model were studied using this method in [37]. This work is divided in the following way. In Section 2, we discuss the properties of the model, in Section 3, we present the SU(3) Schwinger boson formalism, in Section 5 we develop the Kubo formalism of the linear response to calculate the spin conductivity of this model, in Section 5 is dedicated to our conclusions and final remarks.

2. The model

The model that we are interested is represented in Fig. 1 and is defined by the following Hamiltonian:

$$\mathcal{H}=\sum_{<i,j>}J_1(\mathbf{S}_i\cdot\mathbf{S}_j)+\sum_{<<i,j>>}J_2(\mathbf{S}_i\cdot\mathbf{S}_j)+D\sum_i(S_i^z)^2. \quad (1)$$

where $\langle i,j\rangle$ stands for the sum over nearest neighbors and $\langle\langle i,j\rangle\rangle$, mean sum on next nearest neighbors. We consider the value of spin $S=1$. Frustration here is due to the competition between the nearest neighbors and the next nearest neighbors, coupling J_1 and J_2. For sufficiently large values of anisotropies, the system becomes a quantum paramagnet [37,39–41]. The anisotropy forces each spin to be in the nonmagnetic state. The ground state has no long-range magnetic order and there is a finite gap to spin excitations. Decreasing D, the energy gap decreases and goes to zero at a critical D_c, where a quantum phase transition takes place. For $D\le D_c$ and positive, the system is in a gapless phase, that is ordered at $T=0$ in the non-frustrated case.

3. Method

The SU(3) Schwinger boson formalism has been derived to treat systems with single ion anisotropy by Papanicolau [39] being a generalization of the SU(2) formalism. In this formalism we choose the basis:

$$|x\rangle=\frac{i}{\sqrt{2}}(|1\rangle-|-1\rangle),\quad |y\rangle=\frac{1}{\sqrt{2}}(|1\rangle+|-1\rangle),\quad |z\rangle=-i|0\rangle$$

where $|n\rangle$ are eigenstates of S^z. The spin operators are written via a set of three boson operators t_α $(\alpha=x,y,z)$ defined as [37]

$$|x\rangle=t_x^\dagger|v\rangle,\quad |y\rangle=t_y^\dagger|v\rangle,\quad |z\rangle=t_z^\dagger|v\rangle, \quad (2)$$

where $|v\rangle$ is the vacuum state. We also have the constraint condition $t_x^\dagger t_x+t_y^\dagger t_y+t_z^\dagger t_z=1$.

In terms of the t operators we can write

$$S^x=-i(t_y^\dagger t_z-t_z^\dagger t_y),\quad S^y=-i(t_z^\dagger t_x-t_x^\dagger t_z),\quad S^z=-i(t_x^\dagger t_y-t_y^\dagger t_x) \quad (3)$$

the states $t_x^\dagger|v\rangle$ and $t_y^\dagger|v\rangle$, both consist of eigenstates $S^z=\pm1$ and have the average $\langle S^z\rangle=0$. This property will preserve the disorder of the ground state. To study disordered phases, it is convenient to introduce other two bosonic operators $u^\dagger$ and $d^\dagger$ [39]

$$u^\dagger=-\frac{1}{\sqrt{2}}(t_x^\dagger+it_y^\dagger),\quad d^\dagger=\frac{1}{\sqrt{2}}(t_x^\dagger-it_y^\dagger), \quad (4)$$

and so

$$|1\rangle=u^\dagger|v\rangle,\quad |0\rangle=t_z^\dagger|v\rangle,\quad |-1\rangle=d^\dagger|v\rangle, \quad (5)$$

with the constraint $u^\dagger u+d^\dagger d+t_z^\dagger t_z=1$. The spin operators can be also written as [37]

$$S^+=\sqrt{2}(t_z^\dagger d+u^\dagger t_z),\quad S^-=\sqrt{2}(d^\dagger t_z+t_z^\dagger u),\quad S^z=u^\dagger u+d^\dagger d. \quad (6)$$

Schwinger's boson formalism is a mean field approximation that becomes accurate in the $N\to\infty$ limit. For the SU(3) Schwinger boson approach for spins $S=1$, the order parameter has eight components which correspond to the eight generators of the SU (3) group [37]. Substituting Eq. (6) into the Hamiltonian (1) and supposing that the t_z bosons are condensed, i.e. $\langle t_z\rangle=\langle t_z^\dagger\rangle=t$, we obtain [37]

$$\begin{aligned}\mathcal{H}=&\frac{J_1}{2}\sum_{r,\delta}[t^2(d_r^\dagger d_{r+\delta}+u_{r+\delta}^\dagger u_r+u_r d_{r+\delta}+d_r^\dagger u_{r+\delta}^\dagger+h.c.)\\&+(u_r^\dagger u_r-d_r^\dagger d_r)(u_{r+\delta}^\dagger u_{r+\delta}-d_{r+\delta}^\dagger d_{r+\delta})\\&\frac{J_2}{2}\sum_{r,\delta}[t^2(d_r^\dagger d_{r+2\delta}+u_{r+2\delta}^\dagger u_r+u_r d_{r+2\delta}+d_r^\dagger u_{r+2\delta}^\dagger+h.c.)\\&+(u_r^\dagger u_r-d_r^\dagger d_r)(u_{r+2\delta}^\dagger u_{r+2\delta}-d_{r+2\delta}^\dagger d_{r+2\delta})+D\sum_r(u_r^\dagger u_r+d_r^\dagger d_r)\\&-\sum_r\mu_r(u_r^\dagger u_r+d_r^\dagger d_r+t^2-1),\end{aligned} \quad (7)$$

where a temperature dependent chemical potential μ_r is introduced to impose the local constraint $S_r^2=S(S+1)=2$. One solves the Hamiltonian equation (7) using a mean-field approach, replacing the local parameter μ_r by a single parameter μ and making the mean field decoupling for the remaining operators as in the Reference [37]. We make the Fourier transform of the operators u and d and after this, we write the Hamiltonian in a matrix form as [37,38]

$$\mathcal{H}=\frac{1}{2}\sum_k\psi_k^\dagger\mathbf{H}_{\alpha\alpha}\psi_k+E_0, \quad (8)$$

where $\psi_k^\dagger=\left(u_k^\dagger\ d_k^\dagger\ u_{-k}\ d_{-k}\ \tilde{u}_k^\dagger\ \tilde{d}_k^\dagger\ \tilde{u}_{-k}\ \tilde{d}_{-k}\right)$, and

$$\begin{aligned}\mathbf{H}_{\alpha\alpha}=&(\lambda+d)\begin{pmatrix}I&0\\0&I\end{pmatrix}+d\begin{pmatrix}\sigma_x\otimes\sigma_x&0\\0&\sigma_x\otimes\sigma_x\end{pmatrix}\\&+a_k\begin{pmatrix}0&I+\sigma_x\otimes\sigma_x\\0&0\end{pmatrix}+a_k^*\begin{pmatrix}0&0\\I+\sigma_x\otimes\sigma_x&0\end{pmatrix},\end{aligned} \quad (9)$$

where I is the 4×4 identity matrix, 0 is the 4×4 zero matrix and σ_γ; $\gamma=x,y,z$ are the Pauli matrixes that satisfy the rules of

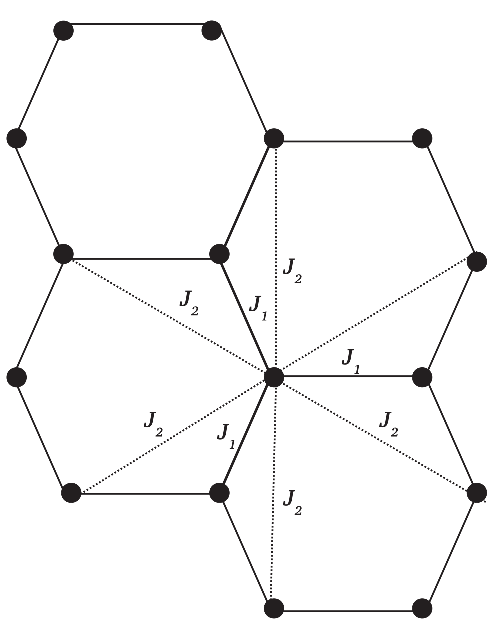

Fig. 1. Representation of the two-dimensional anisotropic frustrated Heisenberg model on a honeycomb lattice with the nearest neighbors and the next nearest neighbors interactions J_1 and J_2.