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On the quantum magnetic oscillations of electrical and thermal conductivities of graphene

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ABSTRACT

Oscillating thermodynamic quantities of diamagnetic materials, specially graphene, have been attracting attention of the scientific community due to the possibility to experimentally map the Fermi surface of the material. These have been the case of the de Haas–van Alphen and Shubnikov–de Haas effects, found on the magnetization and electrical conductivity, respectively. In this direction, managing the thermodynamic oscillations is of practical purpose, since from the reconstructed Fermi surface it is possible to access, for instance, the electronic density. The present work theoretically explores the quantum oscillations of electrical and thermal conductivities of a monolayer graphene under a crossed magnetic and electric fields. We found that the longitudinal electric field can increase the amplitude of the oscillations and this result is of practical and broad interest for both, experimental and device physics.

1. Introduction

The study of graphene is an actual problem of experimental and theoretical condensed matter physics [1], since their two-dimensional lattice and Dirac-like electronic spectrum lead to unique physical properties. As a consequence, emerging phenomena, such as the quantum Hall effect [1–3] and optical absorption [4], just to name some, make graphene promising materials for modern nanoelectronics.

A special attention shall be given to the oscillatory thermodynamic quantities also found on graphene, like the de Haas–van Alphen effect, that rules the oscillations on the magnetization [5,6], the Shubnikov–de Haas effect, that, on its turn, rules the oscillations on the conductivity [7] and, more recently, the oscillating huge and manageable magnetocaloric effect [8–11]. Theoretical and experimental works have been done on this direction and these are indeed of practical interest, since from these oscillations it is possible to access and built up the Fermi surface of the material [12].

The key rule to all of those interesting phenomena is the anomalous Landau Levels (LLs) due to an applied magnetic field; and some works had shown [13–15] that a crossing magnetic and electric fields under the graphene sheet indeed enhance and control a plenty of those emerging phenomena. In this direction, the present work focuses to investigate the oscillations on the electrical and thermal conductivities of a graphene sheet submitted to these crossed fields.

Some well established quantities had been used along the present work, like the energy spectrum of a graphene under a crossed electric $\vec{E}=(E,0,0)$ and magnetic $\vec{H}=(0,0,H)$ fields, that can be written as:

$$\epsilon_{n,p_y}=\mathrm{sgn}(n)\hbar\Omega_c\sqrt{|n|}+v_0p_y, \quad (1)$$

where

$$\Omega_c=(1-\beta^2)^{3/4}\sqrt{2}v_Fl_H^{-1} \quad (2)$$

rules the LLs separation, $\beta=v_0/v_F$, $\vec{v}_0=cE\hat{y}/H$ is the average electron drift velocity perpendicular to the EH plane, $l_H=\sqrt{\hbar c/eH}$ is the magnetic length, $v_F\approx10^8$ cm/s is the Fermi velocity and n is

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an integer. This result can be derived considering both approaches, Dirac formalism [14,15] and the quasi-classical one [13]. The chemical potential had also been used along the present evaluation, and it can be determined from the condition

$$N=-\frac{1}{S}\left(\frac{\partial\Omega}{\partial\mu}\right)_{H,T} \quad (3)$$

where N is the concentration of carriers and S is the graphene area. Taking advantage of the results in reference [16] and neglecting the oscillating part of thermodynamic potential, it is possible to write:

$$\mu=v_F\hbar(1-\beta^2)^{3/4}\sqrt{\pi N} \quad (4)$$

Note thus that the chemical potential also depends on the longitudinal electric field. The condition $\beta=1$ corresponds to the regime in which the electron drift velocity is equal to Fermi velocity; and the electron trajectory on the reciprocal space is no longer closed. In other words, for this critical condition, the orbital motion vanishes, the quantization disappears and the Landau structure collapses.

Next section describes the electrical conductivity, while the following, the thermal one. In few words, we have shown that it is possible to control the quantum oscillations of these conductivities by managing the electric field; and these findings are helpful to guide further experimental works.

2. Electrical conductivity

In order to achieve the oscillations on the electrical conductivity, we have used Kubo's formula, focusing to this purpose on the real part of diagonal conductivity. However, under a quantized magnetic field, Kubo's formula had been generalized by Gorbar et al. [17]; and, for the present case of crossed fields, the conductivity reads as:

$$\sigma(\mu,T,E)=\frac{1}{S}\sum_{p_y}\int_{-\infty}^{\infty}d\epsilon\left(-\frac{\partial f}{\partial\epsilon}\right)A(\epsilon-v_0p_y) \quad (5)$$

where

$$f(\epsilon)=\frac{1}{\exp[(\epsilon-\mu)/k_BT]+1} \quad (6)$$

is the Fermi–Dirac distribution,

$$A(\epsilon)=\frac{4v_F^2\hbar e^2\Gamma^2}{\pi}\sum_n\frac{\left(\epsilon^2+\epsilon_n^2+\Gamma^2\right)}{\left[\left(\epsilon^2-\epsilon_n^2-\Gamma^2\right)^2+4\epsilon^2\Gamma^2\right]}\times\frac{\left(\epsilon^2+\epsilon_{n+1}^2+\Gamma^2\right)}{\left[\left(\epsilon^2-\epsilon_{n+1}^2-\Gamma^2\right)^2+4\epsilon^2\Gamma^2\right]}, \quad (7)$$

$\epsilon_n=\epsilon_{n,p_y}-v_0p_y$ and, finally, Γ is the scattering energy.

Firstly, let us focus our attention to Eq. (7). From the Poisson summation formula

$$\sum_{n=0}^{\infty}F(n)=\frac{F(0)}{2}+\int_0^{\infty}F(x)\,dx+2\,\mathrm{Re}\left\{\sum_{k=1}^{\infty}\int_0^{\infty}F(x)e^{i2\pi kx}\,dx\right\}, \quad (8)$$

Eq. (7) can be represented as a sum of non-oscillating and oscillating parts:

$$A(\epsilon)=A_{no}(\epsilon)+A_{osc}(\epsilon) \quad (9)$$

where the former has two contributions: $A_{no1}(\epsilon)$ and $A_{no2}(\epsilon)$, due to the first and second terms of Eq. (8), respectively. For the sake of clearness, the contribution to the conductivity due to $A_{no1}(\epsilon)$ will be addressed in Appendix A, since this term is much smaller than the other two and, consequently, negligible. Thus, considering $y=\hbar^2\Omega_c^2(x-\mu/\Gamma)$, the second term of $A_{no}(\epsilon)$ can be written as:

$$A_{no2}(\epsilon)=\frac{2e\Gamma^2}{\pi}\frac{c}{(1-\beta^2)^{3/2}H}\int_{-\infty}^{\infty}dy\frac{\left[\epsilon^2+y+\hbar^2\Omega_c^2\frac{\mu}{\Gamma}+\Gamma^2\right]\left[\epsilon^2+y+\hbar^2\Omega_c^2\left(\frac{\mu}{\Gamma}+1\right)+\Gamma^2\right]}{\left\{\left[\epsilon^2-y-\hbar^2\Omega_c^2\frac{\mu}{\Gamma}-\Gamma^2\right]^2+4\epsilon^2\Gamma^2\right\}\left\{\left[\epsilon^2-y-\hbar^2\Omega_c^2\left(\frac{\mu}{\Gamma}+1\right)-\Gamma^2\right]^2+4\epsilon^2\Gamma^2\right\}} \quad (10)$$

where the lower limit of the integral equates to $-\infty$, considering $\mu\gg\Gamma$ (see references [7,17] for further details on this integral limit). The above integrand in the complex plane has two poles of first order in the upper half plane; and using the theory of residues, we obtain the final expression for the second term of the non-oscillating contribution to $A(\epsilon)$:

$$A_{no2}(\epsilon)=\frac{8ec}{(1-\beta^2)^{3/2}H}\frac{|\epsilon|\Gamma\left(\epsilon^2+\Gamma^2\right)}{\left[\hbar^4\Omega_c^4+(4\epsilon\Gamma)^2\right]} \quad (11)$$

The oscillating contribution to $A(\epsilon)$ shall be achieved considering a similar evaluation as above; and after some steps of calculus we obtain:

$$A_{osc}(\epsilon)=\frac{16ec}{(1-\beta^2)^{3/2}H}\frac{|\epsilon|\Gamma(\epsilon^2+\Gamma^2)}{\left[\hbar^4\Omega_c^4+(4\epsilon\Gamma)^2\right]}\times\sum_{k=1}^{\infty}\cos\left[\frac{2\pi k}{\hbar^2\Omega_c^2}(\epsilon^2-\Gamma^2)\right]\exp\left(-\frac{2\pi k}{\hbar^2\Omega_c^2}2|\epsilon|\Gamma\right) \quad (12)$$

Due to the Poisson summation formula, $A(\epsilon)$ could be written with two terms: a non-oscillatory $A_{no}(\epsilon)$ and an oscillatory $A_{osc}(\epsilon)$. Since the electrical conductivity (Eq. (5)) depends on $A(\epsilon)$, it is straightforward to see that the conductivity also has two contributions: a non-oscillatory $\sigma_{no}(\mu,T,E)$ and an oscillatory $\sigma_{osc}(\mu,T,E)$; in such way we can write for the total electrical conductivity:

$$\sigma(\mu,T,E)=\sigma_{no}(\mu,T,E)+\sigma_{osc}(\mu,T,E) \quad (13)$$

It is important to remember that the former term has two contributions; and one of these (σ_{no1}) is described in Appendix A, since it is negligible in comparison to the other term.

From now on, let us focus our attention on the evaluation of these two terms: $\sigma_{no}\approx\sigma_{no2}$ and σ_{osc}. First, Eq. (5) shall be simplified; and, to go further, let us consider:

$$\sum_{p_y}=\frac{L_y}{\pi\hbar}\int_0^{p_{ymax}}dp_y, \quad (14)$$

where p_{ymax} is determined from the condition [15]:

$$0<x=\frac{c}{eH}p_y<L_x, \quad (15)$$

that leads to $p_{ymax}=eHL_x/c$.

Considering the low temperature limit $\mu\gg k_BT$, as well as Eq. (11) into Eq. (5), we obtain:

$$\sigma_{no2}(\mu,T,E)=\frac{8ec}{(1-\beta^2)^{3/2}\pi\hbar HL_x}\int_0^{p_{ymax}}dp_y\frac{\Gamma(\mu-v_0p_y)\left[(\mu-v_0p_y)^2+\Gamma^2\right]}{\left\{\hbar^4\Omega_c^4+[4\Gamma(\mu-v_0p_y)]^2\right\}} \quad (16)$$