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Electron–phonon heat exchange in layered nano-systems

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ABSTRACT

We analyze the heat power P from electrons to phonons in thin metallic films deposited on free-standing dielectric membranes in a temperature range in which the phonon gas has a quasi two-dimensional distribution. The quantization of the electrons wavenumbers in the direction perpendicular to the film surfaces lead to the formation of quasi two-dimensional electronic sub-bands. The electron–phonon coupling is treated in the deformation potential model. If we denote by T_e the electrons temperature and by T_{ph} the phonons temperature, we find that $P \equiv P^{(0)}(T_e) - P^{(1)}(T_e, T_{ph})$.

Due to the quantization of the electronic states, both $P^{(0)}$ and $P^{(1)}$, plotted vs (T_e, d) show very strong oscillations with d, forming sharp crests almost parallel to T_e. From valley to crest, both $P^{(0)}$ and $P^{(1)}$ increase by more than one order of magnitude. In the valleys between the crests, $P \propto T_e^{3.5} - T_{ph}^{3.5}$ in the low temperature limit, whereas on the crests P does not have a simple power law dependence on temperature. The strong modulation of P with the thickness of the film may provide a way to control the electron-phonon heat power and the power dissipation in thin metallic films. On the other hand, the surface imperfections of the metallic films can smoothen these modulations.

1. Introduction

In a recent paper Nquyen et al. [1] reported remarkable cooling properties of normal metal-insulator-superconductor (NIS) tunnel junctions refrigerators, by reaching electronic temperatures of 30 mK or below, from a bath temperature of 150 mK, at a cooling power of 40 pW. Such micro-refrigerators have great potential for applications, since they can be mounted directly on chips for cooling qubits or ultra-sensitive detectors, like micro-bolometers or micro-calorimeters.

The principle of operation of NIS micro-refrigerators has been explained in several publications (e.g. [2–7]) and consists basically in cooling of a normal metal island by evacuating the "hot" electrons (from above the Fermi sea) into a superconductor while injecting "cold" electrons (below the Fermi sea) from another superconductor using a pair of symmetrically biased NIS tunnel junctions. If the normal metal island is deposited on a chip, then it can serve as a refrigerator by cooling the chip through electron–phonon interaction. The efficiency of the electron cooling process is controlled by the bias voltages of the NIS junctions, whereas the success of the chip refrigeration strongly depends on the electron–phonon coupling. Moreover, due to the strong temperature dependence of the electric current through the junctions at fixed bias voltage (or the strong variation of voltage with temperature at fixed current) the NIS junctions can also serve as thermometers. Because of this, if the normal metal island absorbs radiation, the device turns into a very sensitive radiation detector [5,8].

When it works as a detector, the normal metal island can be kept at the nominal working temperature either by cooling it directly through NIS junctions (eventually through the thermometer junctions), or indirectly through electron–phonon coupling to a cold substrate [9–11]. Therefore in any situation, i.e. when the NIS junctions work as coolers, thermometers, or radiation detectors, the electron–phonon coupling plays a central role in the functionality of the device.

A typical experimental setup [9,5] is depicted in Fig. 1 and consists of a Cu film of thickness of the order of 10 nm deposited on a dielectric silicon-nitride (SiN_x) membrane of thickness of the order of 100 nm. The Cu film is the normal metal island and is connected to superconducting Al leads through NIS tunnel junctions. When it is used as a radiation detector, to reach the sensitivity required by astronomical observations, the working temperature of the device should be in the range of hundreds of mK or below [1,10,11]. At such temperatures the phonon gas in the layered structure formed by the normal metal island and the supporting membrane undergoes a dimensionality cross-over from a three-dimensional (3D) gas (at higher temperatures) to a quasi two-dimensional (2D) gas (at lower temperatures) [12–14].

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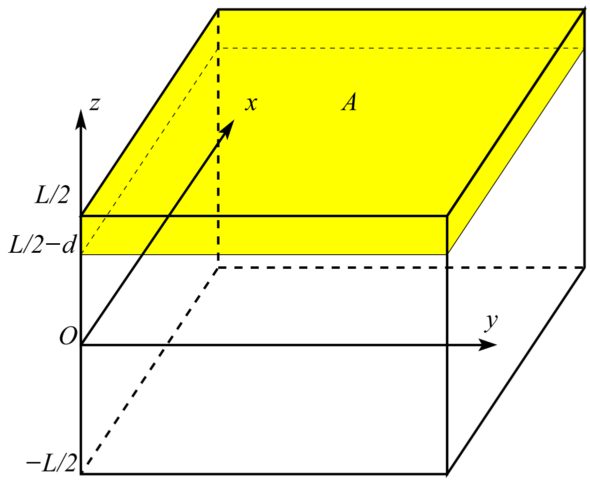

Fig. 1. (Color online). The schematic model of the system. On a dielectric membrane with parallel surfaces perpendicular on the z axis is deposited an uniform metallic film. The surfaces of the membrane cut the z axis at $-L/2$ and $L/2-d$, whereas the upper surface of the film cuts the z axis at $L/2$. In the (x,y) plane the area of the device is A ($\gg L^2$). The whole system is treated like a homogeneous elastic continuum, with the elastic properties and the density of silicon nitride (SiN_x).

In the stationary regime, one may assume that the electrons in the metallic layer have a Fermi distribution characterized by an effective temperature T_e, whereas the phonons have a Bose distribution of effective temperature T_{ph}. In 3D bulk systems the heat flux between the electrons and phonons varies as $T_e^5 - T_{ph}^5$ at low temperatures (see e.g. [15]). Such a model is not justified for our devices and finite size effects [16] have to be taken into account.

Phenomenologically, a number of experimental studies have been interpreted by assuming that the heat flux has a temperature dependence proportional to $T_e^x - T_{ph}^x$, where $x < 5$ [17–19]. On the other hand, a theoretical investigation of the surface effects for a thin metallic film deposited on a half-space (semi-infinite) dielectric showed that the value of x is actually larger than 5 [20]. Qualitatively, the growth of the exponent x in the lowest part of the measured temperature range has been later found in some experiments when metallic films were deposited on bulky substrates [18].

The electron scattering rate caused by interaction with 2D (Lamb) phonon modes in a semiconductor quantum well (QW) has been studied in Refs. [21,16] and in a double heterostructure QW including the piezoelectric coupling in Ref. [22].

The electron–phonon heat transfer in monolayer and bilayer graphene was studied in Ref. [23] and a temperature dependence of the form $T_e^4 - T_{ph}^4$ was found in the low temperature regime. For a quasi one-dimensional geometry (metallic nanowire) the electron–phonon power flux was studied theoretically in [24] and a T^3 dependence was obtained. It is argued that a general temperature dependence of the form $T_e^{s+2} - T_{ph}^{s+2}$ should be valid, where s is the smaller of dimensions of the electron and phonon system [23].

In this context it is interesting to analyze theoretically the temperature dependence of the heat transferred between electrons and phonons in the typical experimental setup of Fig. 1. We assume that the metallic layer is made of Cu and the supporting membrane is silicon nitride (SiN_x). We first observe that the dimensionality crossover of the phonon gas in a 100 nm thick SiN_x membrane occurs around a scaling temperature $T_C \equiv c_l\hbar/(2k_BL) \approx$ 237 mK [14]. Below this temperature one may expect a quasi-2D behavior of the phonon system (rigorously, for $T \ll T_C$), whereas above it (rigorously, at $T \gg T_C$) a 3D model would suffice.

We carry out our analysis employing a QW picture of the metallic film [21,16,22,25] by taking into account the discretization of the components of the electrons wavenumbers perpendicular to the membrane's surfaces. We work in a temperature range $T < 200$ mK in which the phonon gas is quasi-2D and we observe that the electron–phonon heat flux P cannot be simply described by a single power-law dependence, $P \propto T_e^x - T_{ph}^x$. While in some ranges of d we have $P^{(0)} \propto T_e^{3.5} - T_{ph}^{3.5}$, in general the heat flux has a very strong oscillatory behavior as a function of d. At certain regular intervals the power flux increases sharply with the film thickness by at least one order of magnitude and in the regions of increased heat flux the exponent of the temperature dependence is not well defined.

2. Electron–phonon interaction

2.1. The electron gas

The electron system is described as a gas of free fermions confined in the metallic film (Fig. 1). The electron wavevector will be denoted by $\mathbf{k} \equiv (\mathbf{k}_\parallel, k_z)$, where $\mathbf{k}_\parallel$ and k_z are the components of $\mathbf{k}$ perpendicular and parallel to z, respectively. The wavefunctions satisfy the Dirichlet boundary conditions on the film surfaces (at $L/2-d$ and $L/2$) and periodic boundary conditions in the (xy) plane, $\psi_{\mathbf{k}_\parallel,n}(\mathbf{r},t) = \phi_n(z)e^{i(\mathbf{k}_\parallel\cdot\mathbf{r}_\parallel - \epsilon_{\mathbf{k}_\parallel,n}t/\hbar)}/\sqrt{A}$, where $n = 1,2,\ldots$, $k_z = \pi n/d$, $\phi_n(z) = \sqrt{2/d}\,\sin\left[(z+d-\frac{L}{2})k_z\right]$, and A is the area of the device surface. The electron energy is $\epsilon_{\mathbf{k}} = \frac{\hbar^2k^2}{2m_e} = \frac{\hbar^2k_\parallel^2}{2m_e} + \frac{\hbar^2\pi^2}{2m_ed^2}n^2 \equiv \epsilon_{k_\parallel,n}$, where m_e is the electron mass, $k \equiv |\mathbf{k}|$, and $k_\parallel \equiv |\mathbf{k}_\parallel|$. We denote the highest electronic sub-band populated at 0 K by ϵ_{n_F}, where $n_F \equiv \left\lfloor \frac{\sqrt{2m_eE_F}}{\pi\hbar}d \right\rfloor$ ($\lfloor x \rfloor$ is the biggest integer smaller than x).

2.2. The phonon gas

For simplicity, we treat the whole system – supporting membrane and metallic film (Fig. 1) – as a single elastic continuum of thickness L, area A, and volume $V_{ph} = LA$. The elastic modes of such a system [16,14,26,27] have the form $\mathbf{w}_{\mathbf{q}_\parallel\xi}(z)e^{i(\mathbf{q}_\parallel\cdot\mathbf{r}_\parallel - \omega_{\mathbf{q}_\parallel\xi}t)}/\sqrt{A}$, where $\mathbf{q}_\parallel$ and $\mathbf{r}_\parallel$ are the components of the wavevector and position vector, respectively, parallel to the (x,y) plane. The functions $\mathbf{w}_{\mathbf{q}_\parallel\xi}(z)$ are normalized on the interval $z \in [-L/2, L/2]$, namely $\int_{-L/2}^{L/2} \mathbf{w}_{\mathbf{q}_\parallel\xi}(z)^\dagger \mathbf{w}_{\mathbf{q}_\parallel\xi'}(z)dz = \delta_{\xi,\xi'}$.

The elastic modes are divided into three main categories: horizontal shear (h), dilatational or symmetric (s) with respect to the mid-plane, and flexural or antisymmetric (a) with respect to the mid-plane. The quantization of the elastic modes in the z direction leads to the formation of phonon branches (or sub-bands), such that a phonon mode is identified by its symmetry $\alpha = h, s, a$, sub-band number $\nu = 1,2,\ldots$, and $\mathbf{q}_\parallel$; ξ represents the pair (α,ν).

The h modes are simple transversal modes. The free boundary conditions imposed at the upper and lower surfaces of the membrane quantize the z-component of the wavevector at the values $q_{thn} = n\pi/L$, $n = 0,1,\ldots$, whereas $q_{lhn} \equiv 0$. If we denote by $q_\parallel \equiv |\mathbf{q}_\parallel|$, then the phonon frequency is $\omega_{q_\parallel,h,\nu} = c_t\sqrt{q_\parallel^2 + q_{t,h,\nu}^2}$, where by c_t and c_l we denote the transversal and longitudinal sound velocities, respectively. The sound velocities are determined by the Lamé coefficients λ and μ, and the density ρ of SiN_x: $c_t = \mu/\rho$, and $c_l = (\lambda + 2\mu)/\rho$.

The s and a modes are superpositions of longitudinal and transversal modes, oscillating in a plane perpendicular to the surfaces. The quantization relations for $q_{l\alpha}$ and $q_{t\alpha}$ – which are the z components of the wavevectors corresponding to the