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Drag on a single fluid sphere translating in power-law liquids at moderate Reynolds numbers

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Abstract

A numerical investigation has been carried out to obtain the steady state drag coefficients and flow patterns of a single Newtonian fluid sphere sedimenting in power-law liquids. A finite difference method based simplified marker and cell (SMAC) algorithm has been implemented on a staggered grid arrangement to solve the continuity and momentum equations. For both phases, the convective terms have been discretized using the quadratic upstream interpolation for convective kinematics (QUICK) scheme, and diffusive and non-Newtonian terms with central differencing scheme. An exponential transformation has been applied in the radial direction for the continuous phase computational domain. In order to ensure the accuracy of the solver, extensive validation has been carried out by comparing the present results with the existing literature values for a few limiting cases. Further, in this study the effects of the Reynolds number (Re_o), internal to external fluid characteristic viscosity ratio (k) and power-law index (n_o) on the continuous phase flow field, pressure drag (C_{dp}), friction drag (C_{df}) and total drag (C_D) coefficients have been analyzed over the range of parameters: $5 \leqslant Re_o \leqslant 500$, $0.1 \leqslant k \leqslant 50$ and $0.6 \leqslant n_o \leqslant 1.6$. Based on numerical results obtained in this work, a simple correlation has been proposed for the total drag coefficient, which can be used to predict the rate of sedimentation of a fluid sphere in power-law liquids.

Keywords: Fluid sphere; Drag; Power-law liquids; Viscosity ratio

1. Introduction

Owing to their overwhelming theoretical and pragmatic significance, in recent years considerable effort has been directed at investigating the drag behaviour of single bubbles and rigid spheres in a great variety of non-Newtonian fluids including purely viscous, viscoplastic and viscoelastic fluids. An inspection of the recent reviews of the pertinent literature (Chhabra, 2006) shows that the sedimentation behaviour of rigid particles in purely viscous fluids (mostly power-law) has been studied most thoroughly, followed by that in viscoplastic and in viscoelastic liquids. Based on a combination of the numerical and experimental studies, reliable results are now available on drag coefficient—Reynolds number behaviour for a sphere falling in unconfined fluids up to about Reynolds number of 1000, which is nowhere near the wide range of conditions encompassed by the corresponding literature for Newtonian fluids (Clift et al., 1978; Michaelides, 2006). At the other extreme is the case of gaseous bubbles rising freely in non-Newtonian fluids. Here too, based on a combination of analytical and/or numerical results coupled with experimental data, reliable information on their drag behaviour in power-law fluids is now available up to about the Reynolds number of 500 (Rodrigue, 2001a,b, 2004; Chhabra, 2006; Dhole et al., 2007). However, it is appropriate to add here that the behaviour of rising gas bubbles in non-Newtonian liquids differs significantly from that of solid spheres at one hand and that of bubbles in Newtonian fluids at the other hand, as noted in detail elsewhere (Clift et al., 1978; Chhabra, 2006). These differences stem from the ability of the bubble to deform thereby exhibiting a great variety of shapes determined by the net resultant forces acting on the bubbles.

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Furthermore, the surface active agents also influence the bubble dynamics in a significant manner (Thorsen et al., 1968).

In contrast, much less is known about the intermediate case of a liquid droplet undergoing steady translation in a quiescent power-law continuous medium. Typical examples entailing the translation of a droplet in non-Newtonian continuous phase include the use of polymer solutions in enhanced oil recovery operations wherein the oil droplets become suspended in polymer solutions and/or during their pipeline transport in the form of oil-in-polymer solution type emulsions, or in scores of personal care products (lotions, shampoos, creams), food-stuffs, pharmaceutical products wherein polymeric thickening agents are used to enhance their stability for extended shelf-life. Irrespective of whether the separation in such emulsions is desirable or not, the need to estimate the free settling velocity of a drop in power law continuous medium arises frequently in process design calculations. From a theoretical standpoint, this case differs from that of a rigid sphere and of a gas bubble in so far that one needs to solve the governing equations for the continuous phase only in the latter cases. On the other hand, for a fluid sphere, one needs to solve the governing equations for both inner and outer phases which are coupled via the continuity of the velocity and tangential stress at the interface. Due to the highly non-linear form of the viscous terms, theoretical solution is not possible even for the slow translation (at vanishingly small values of the Reynolds number) of a Newtonian fluid sphere in the simple power-law continuous phase, when the non-linear intertial terms in momentum equations are altogether neglected. Therefore, the progress in this area has been rather slow from theoretical and numerical stand points. This work aims to elucidate the role of power-law rheology on the drag behaviour of a Newtonian fluid sphere undergoing steady translation in a quiescent power-law continuous phase over a wide range of the Reynolds number and of the ratio of the viscosity of the two phases. It is, however, appropriate to begin with a short review of the previously available scant literature on this subject.

2. Previous work

Ever since the celebrated analysis of Hadamard (1911) and Rybzynski (1911) for the creeping motion of a fluid sphere in an unconfined incompressible Newtonian fluid, considerable research effort has been extended in studying the drag behaviour of Newtonian-spherical and non-spherical-drops in another immiscible Newtonian medium focusing on a variety of aspects including the drag coefficient–Reynolds number relationship, wall effects, prediction of shape, wake dynamics, etc. Consequently over the years, an extensive body of information has accrued which has been reviewed by many authors including Clift et al. (1978), Zapryanov and Tabakova (1999), Michaelides (2006), etc. Suffice it to add here that reliable schemes to predict the rates of momentum, heat and mass transfer characteristics from single drops sedimenting in Newtonian continuous media are now available.

In contrast, very little information is available for the analogous flow when the continuous phase is non-Newtonian (Chhabra, 2006). Owing to the non-linearity of the viscous term, governing equations are highly non-linear even when the inertial effects are neglected and only the creeping motion is considered. Therefore, an analytical solution, akin to the Hadamard–Rybzynski expression, is not possible even for the simple power-law model. Consequently, only approximate analyses are available even in the zero–Reynolds number limit. Early analyses such as that of Nakano and Tien (1968),Mohan et al. (1972), Mohan (1974), Mohan and Venkateswarlu (1976), Jarzebski and Malinowski (1986, 1987a,b), Chhabra and Dhingra (1986), etc., are based on the use of the velocity and stress variational principles. This approach yields upper and lower bounds on the drag force (and hence on the terminal velocity); usually the two bounds diverge with the increasing degree of shear-thinning behaviour. In the absence of any definitive information, the use of the arithmetic mean of the two bounds is suggested. It is useful to recall here that this approach is not only restricted to the inertialess flow in shear-thinning fluids, but it yields true upper and lower bounds only for Newtonian and power-law fluids. In spite of this limitation, this approach has been used for Ellis (Mohan and Venkateswarlu, 1976) and Carreau model fluids (Chhabra and Dhingra, 1986, 1988; Jarzebski and Malinowski, 1987b). However, this approach predicts a slight enhancement in the value of drag coefficient for a Newtonian droplet settling in a quiescent power-law medium ($n < 1$) as compared to that in a Newtonian continuous phase otherwise under identical conditions. Beyond the creeping flow regime, indeed very little is known about the effect of the power-law rheology on drop motion at finite Reynolds numbers. As far as known to us, only Nakano and Tien (1970) have studied the steady translation of a Newtonian drop in power-law fluids up to the Reynolds number, $5 \leqslant Re_o \leqslant 25$, the characteristic viscosity ratio (of the dispersed to the continuous phase), $0.01 \leqslant k \leqslant 2$ and the power-law index, $0.6 \leqslant n_o \leqslant 1$. However, their numerical results for Newtonian fluids differ significantly from the literature values (e.g., Feng and Michaelides, 2001) thereby casting some doubt on the accuracy of their results. This uncertainty seems to stem from the inadequacy of the numerics used in their study. The case of a power-law sphere undergoing steady translation in a Newtonian and in a power-law continuous phase has been, respectively, treated by Gurkan (1989) for finite Reynolds numbers and by Tripathi and Chhabra (1994) in the creeping flow regime.

On the other hand, many investigators including Mhatre and Kintner (1959), Fararoui and Kintner (1961), Mohan et al. (1972), Acharya et al. (1978), Chhabra and Bangun (1997) and Rodrigue and Blanchet (2001) have reported experimental results on the settling velocity of freely falling Newtonian drops in power-law and in viscoelastic liquids. Shirotsuka and Kawase (1973) have investigated the effect of power-law index on the motion and mass transfer of fluid spheres in non-Newtonian systems at high Reynolds and at high Peclet numbers by obtaining the approximate expressions for the stream functions of both phases. Wanchoo et al. (1999) have reported the rise or fall of liquid drops in Newtonian media and presented a correlation for drag coefficient. Subsequently, they have also studied the shapes of liquid drops during their free fall in