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Quasi-direct numerical simulation of lift force-induced particle separation in a curved microchannel by use of a macroscopic particle model

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Abstract

The macroscopic particle model (MPM) based on the finite volume method is employed to validate a mechanism of lift force-induced particle separation in a curved microchannel. According to the particle velocity at each time step in the unsteady simulation, the MPM gives momentum to those fluid cells touching the particle physical boundary. The summation of the given momentum with the reversed sign is divided by the time step to obtain the hydrodynamic force acting on the particle. Namely, the existence and motion of the particle causes fluid flow around the particle, while the flow field caused by the particle determines the particle motion by means of the hydrodynamic force. Therefore, the MPM can be regarded as implementing a quasi-direct numerical simulation over the static computational cells. The lift force acting on a spherical particle in a shear flow is a purely hydrodynamic force caused by the flow field around the particle. It is expected, therefore, that the MPM could predict the lift force effect without any additional model. At first, it is shown that the MPM is capable of predicting particle migration away from the wall of a straight microchannel due to the lift force. In a curved microchannel, subsequently, the particle trajectories from representative release points predicted by the MPM are compared to those predicted by a common particle tracking method without any lift force model. The MPM predicted that the particle trajectories are confined in the outer region of the channel cross-section. On the other hand, the circulating trajectories predicted by the tracking method tend to expand due to centrifugal force caused by the Dean vortices. It is concluded, therefore, that the lift force due to the steep shear rate is a significant factor to cause particle separation in a curved microchannel.

Keywords: Lift force; Particle separation; Microchannel; CFD; Quasi-direct numerical simulation

1. Introduction

A microseparator/classifier is a novel microdevice whose functional part is a curved microchannel with rectangular cross-section (Ookawara et al., 2003, 2004a,b 2005b). The width and depth of the prototype devices fall into the range of 100–400 μm. The curvature radius is always set at 20 mm and the arc angle is varied from 30 to 180°. The arc is connected to a straight microchannel at one end, while it symmetrically bifurcates at the other end. A feed slurry is fed into the straight channel and particles are collected from the outer branch after passing through the arc where a particle concentration profile develops. It has been experimentally proven that the device shows superior separation efficiency in terms of both classification sharpness and required energy for a desired cut size compared to large-scale hydrocyclones (Ookawara et al., 2005b). It has been argued that these superior features are brought about by the lift force due to the steep shear rates in microchannels in a series of numerical studies (Ookawara et al., 2004c, 2005c, 2006).

The lift force is a hydrodynamic force acting on a spherical particle in a shear flow. The most common correlation for the magnitude is given by Saffman (1965, 1968):

$$F_{L,\mathrm{Saf}} = 1.615\rho\nu^{1/2}d_p^2(u-v)\left|\frac{\mathrm{d}u}{\mathrm{d}y}\right|^{1/2}\mathrm{sign}\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right), \tag{1}$$

where ρ and ν are the fluid density and kinematic viscosity, d_p is the diameter of the spherical particle, u and v are the velocities

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of the fluid and the particle in the x- direction and $\mathrm{d}u/\mathrm{d}y$ is the shear rate of the uniform flow field. The correlation requires the following conditions to be satisfied:

$$Re_S = \frac{|u-v|d_p}{\nu} \ll 1, \tag{2}$$

$$Re_G = \frac{|\mathrm{d}u/\mathrm{d}y|d_p^2}{\nu} \ll 1, \tag{3}$$

$$Re_\Omega = \frac{\Omega d_p^2}{\nu} \ll 1, \tag{4}$$

and

$$\varepsilon = \frac{Re_G^{1/2}}{Re_S} \gg 1, \tag{5}$$

where Ω is the rotational speed of the particle.

For instance, a spherical particle with density and diameter of $1190\,\mathrm{kg/m^3}$ and $20\,\mu\mathrm{m}$ in water ($995.65\,\mathrm{kg/m^3}$, $0.0007973\,\mathrm{Pa\,s}$, $30\,^\circ\mathrm{C}$) can be well separated by a curved microchannel with width, depth and curvature of 200, 150 μm and 20 mm at Reynolds number 690 (Ookawara et al., 2005b). The flow condition gives a mean velocity of 3.2 m/s and consequently a centrifugal acceleration of $520\,\mathrm{m/s^2}$. The centrifugal slip velocity can be calculated as 2.8 mm/s based on Stokes' law. If the slip velocity can be regarded as a characteristic velocity, the particle Reynolds number (Re_S) of 0.070 well satisfies Saffman's first constraint. By using an equivalent diameter D_h and mean velocity U, on the other hand, a representative shear rate might be calculated as $8U/D_h = 1.5 \times 10^5\ 1/\mathrm{s}$. The expression of $8U/D_h$, which gives a shear rate on the wall in laminar flow conditions for a circular pipe, is expediently utilized for the discussion. The particle Reynolds number (Re_G) based on the shear rate takes the value of 75, which is far beyond the second constraint of Saffman's lift force model. The shear rate always takes a maximum value at the channel wall and it decreases nearer the high-velocity region around the channel axis. Saffman's lift force model apparently cannot be applied near the wall, although there might exist a region where it can be applied. As the channel Reynolds number increases, the dimensions of applicable region are reduced near the axis while the infringement of the constraint becomes severe near the wall.

There are several well-known correlations to overcome the constraints of Saffman's lift force model. Mei (1992) proposed the following two correlations for a finite Re_S ($0.1 \leqslant Re_S \leqslant 100$):

$$\frac{F_{L,\mathrm{Mei}}}{F_{L,\mathrm{Saf}}} = (1 - 0.3314\alpha^{1/2})\exp\left(-\frac{Re_S}{10}\right) + 0.3314\alpha^{1/2} \quad (Re_S \leqslant 40), \tag{6}$$

$$\frac{F_{L,\mathrm{Mei}}}{F_{L,\mathrm{Saf}}} = 0.0524(\alpha Re_S)^{1/2} \quad (Re_S > 40), \tag{7}$$

and finite shear rate,

$$\alpha = \tfrac{1}{2} Re_S \varepsilon^2 \tag{8}$$

($0.005 \leqslant \alpha \leqslant 0.4$). The above characteristic values for the microseparator/classifier give the α value of 540. The Re_S is somewhat smaller compared to the considered ranges while α is considerably larger than the limits.

Further, the following expression was given by McLaughlin (1991) for $0.1 \leqslant \varepsilon \leqslant 20$:

$$\frac{F_{L,\mathrm{McL}}}{F_{L,\mathrm{Saf}}} = 0.3\{1 + \tanh[2.5\log(\varepsilon + 0.191)]\}\{0.667 + \tan[6(\varepsilon - 0.32)]\}. \tag{9}$$

Since the ε can be calculated as 120 for the present separator, this correlation cannot be applicable in the present case. This is caused by the extremely high shear rate that can be found in the microchannels. McLaughlin (1991) also proposed the following correlations for a wider ε range:

$$\frac{F_{L,\mathrm{McL}}}{F_{L,\mathrm{Saf}}} = 1 - \frac{0.287}{\varepsilon^2}, \quad \varepsilon \gg 1, \tag{10}$$

$$\frac{F_{L,\mathrm{McL}}}{F_{L,\mathrm{Saf}}} = -140\varepsilon^5 \ln\left(\frac{1}{\varepsilon^2}\right), \quad \varepsilon \ll 1. \tag{11}$$

Although the ε value of 120 for the microseparator/classifier satisfies the condition of ($\varepsilon \gg 1$), McLaughlin's expressions also require that both Re_S and Re_G are small compared to unity. As mentioned above, the Re_G does not satisfy the constraint near the wall and therefore this expression is also not applicable for the present device.

Further, all the above correlations have been derived and proposed assuming an infinite flow field in one direction and with a constant velocity gradient. The assumption can be locally satisfied when the particle is considerably smaller than the dimensions of the device. In microsystems, however, the particle diameter is often comparable in size to the channel width. When the velocity gradient can be varied within the scale of particle, the applicability of the correlations is not always assured. Moreover, in the curved microchannel of microseparator/classifier, there appear Dean vortices as secondary flow pattern. The validity of the correlations has not been investigated in such a complicated flow field.

The previous numerical works showed that the lift force was an indispensable factor to reasonably predict the concentration effects that correspond to the experimental results. However, the adopted lift force model (Drew and Lahey, 1993), based on Euler–Euler approach, was modified by the authors in terms of the acting direction and magnitude (Ookawara et al., 2004c, 2005c, 2006). Therefore, there is a need to validate the mechanism of lift force-induced particle separation in the microseparator/classifier by means of a different approach to support the previous studies. The direct numerical simulation associated with dynamic meshing around a moving particle is expected to be the most accurate method for this purpose. Although any hydrodynamic effect such as drag and lift will be always predicted automatically without any assumption by the direct