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# Shot noise in superconducting wires with a periodic modulation of the chemical potential

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ABSTRACT

We investigate the shot noise in superconducting wires under the periodic modulation of the chemical potential. The nonequilibrium Green's function technique is employed, and the formula for current and shot noise is obtained. The coupling between the Majorana bound states at ends of wire can be tuned by the periodic modulation of chemical potential. It is related with the strength $A$ and the phase $\delta$ intimately. The current, shot noise and the corresponding Fano factor display oscillation behavior as the strength $A$ increases. In addition, the coupling between Majorana bound states can be suppressed by strong coupling between leads and superconducting wire.



## 1. Introduction

In 1937, Ettore Majorana found a real solution to the Dirac equation [1]. In contrast to the traditional solutions of the Dirac equation where $c \neq c^\dagger$, this real solution equals to the antiparticle solution $\gamma = \gamma^\dagger$. The Majorana fermion has the anticommunication relation $\{\gamma_i, \gamma_j^\dagger\} = 2\delta_{ij}$ in contrast to a Dirac fermion which obeys the $\{c_i, c_j^\dagger\} = \delta_{ij}$. Due to the Pauli exclusion principle, the Dirac fermion obeys $c^2 = 0$, however the Majorana fermion obeys $\gamma^2 = 1$. Due to these facts, a Majorana fermion does not have the usual fermionic statistics but it follows non-Abelian statistics which makes the Majorana fermion particularly interesting in the field of topological quantum computation [2]. Therefore, searching for Majorana fermions in the condensed matter system has attracted intensive studies in past years [3–7], and various schemes for realizing Majorana fermions in solid state systems have been proposed [8]. These systems include p-wave superconductors [9], spin–orbit-coupled quantum wells [10–12], topological insulators [13,14], nanowires [15–18] and half metals [19,20]. In order to verify the existence of Majorana fermions in these system, transport behaviors have been investigated, including measurement of noise [21,22], differential conductance [23,24], resonant Andreev reflection [25] and periodic Majorana–Josephson currents [15,16].

Among these proposals, one-dimensional wire with strong spin–orbit coupling and Zeeman fields is believed to host Majorana fermions at the two ends [15,16,26]. The experimental finding of a zero-bias peak [27] of conductance in this one-dimensional wire is in precise agreement with the theoretical predictions [28,29] and it can be taken as the signature of Majorana bound states [23,24]. The zero-bias peak of conductance is related with the temperature, tunnel barrier potential, magnetic field and disorder of nanowire [30,31]. Recently, Wu et al. investigated the behaviors of shot noise in this topological superconducting wire between two normal leads by nonequilibrium Green's functions method for tight-binding model [32]. The Fano factor for single-lead devices reaches 2 at low bias due to the resonant Andreev reflection, it is significantly reduced by a finite though small coupling to the second lead. These phenomena are clear signatures of Majorana bound states. In order to control the physical properties of nanowires, we modulate the chemical potential periodically through the gate. The Majorana edge states in the p-wave superconducting wire are robust against the periodical modulation [33].

Motivated by the works of Wu et al. [32] and Chen et al. [33], we investigate the shot noise in superconducting wires with a periodic modulation of the chemical potential in this paper. The differential conductance and shot noise are influenced by the periodic modulation of the chemical potential intimately. The two-peak structure in differential conductance and the valley in Fano factor show that the zero energy state is split by the coupling between the Majorana

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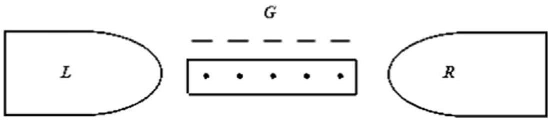


**Fig. 1.** The quantum wire of topological superconductor sandwiched between two normal leads. The periodic modulated chemical potential can be realized through the control of gates.

bound states. The two-peak structure and the valley disappear gradually due to the coupling decreases gradually until disappear as the chemical potential $A$ increases. It indicates that the coupling between end states can be destroyed by stronger modulation of the chemical potential. However, the destroyed coupling can recover as the phase $\delta$ changed. In addition, the current and shot noise display oscillation behavior under the strong modulation.

This paper is organized as follows. In Section 2, we present the Hamiltonian of our system and the detail algebraic expressions for shot noise and current by using NGF technique and Wick theorem. The numerical results are given in Section 3 with analyses. Finally, a summary is made in Section 4.

## 2. Model and formalism

The device under our consideration is composed of two normal leads coupled the one-dimensional quantum wire (Fig. 1). We consider the circumstance where the two leads are biased by the dc voltage $V$ which is the drop of chemical potentials between two leads $\mu_L-\mu_R=eV$. The center nanowire is in proximity to a conventional s-wave superconductor and subject to a Zeenman field, and a finite pair potential $\Delta$ is induced. When the Zeeman splitting energy $V_z$, the proximity-induced order parameter $\Delta$, and the chemical potential $\mu$ satisfy the condition $V_z>\sqrt{\Delta^2+\mu^2}$, the nanowire is driven into a topological superconducting phase, and a pair of zero-energy MBSs will emerge at the ends of the nanowire [10,15]. The Hamiltonian of the nanowire can be given in tight-binding form as follows [34]:

$$H_{wire}=\sum_{j=1}^{N}\Big[-\frac{t}{2}(c_j^\dagger c_{j+1}+h.c.)-(\mu_j-t)c_j^\dagger c_j-\frac{\alpha_{SO}}{2}(ic_j^\dagger\sigma_y c_{j+1}+h.c.)+V_z c_j^\dagger\sigma_z c_j+\Delta(c_{j\uparrow}c_{j\downarrow}+h.c.)\Big] \tag{1}$$

where $c_j^\dagger(c_j)$ is the creation (annihilation) operator of fermions at the $j$th site, $t$ is the nearest-neighbor hopping amplitude, $\alpha_{SO}$ characterizes the spin–orbit coupling effect in the tight-binding model, and $\sigma_{y,z}$ are the corresponding Pauli matrices. The modulated chemical potential $\mu_j$ is given by

$$\mu_j=A\cos(2\pi j\alpha+\delta) \tag{2}$$

with $A$ being the strength, $\alpha$ a rational number, and $\delta$ an arbitrary phase shift. The modulated chemical potential can be realized through the control of gates for the quantum wires [35,36]. Under modulation of the periodic spatial modulation of the gate voltage, the energy spectrum of the quantum wire acquires a periodic charge-density wave.

The Hamiltonian of electrons in the leads is $H_{leads}=\sum_{\gamma,k\sigma}\varepsilon_{\gamma,k\sigma}c_{\gamma,k\sigma}^\dagger c_{\gamma,k\sigma}$, where the electron creation (annihilation) operators in the leads are denoted by $c_{\gamma,k\sigma}^\dagger(c_{\gamma,k\sigma})$. The coupling Hamiltonian of the nanowires to the leads is given by

$$H_{leads-wire}=\sum_{k,\sigma}[(T_L c_{L,k\sigma}^\dagger c_{1\sigma}+T_R c_{R,k\sigma}^\dagger c_{N\sigma})+h.c.] \tag{3}$$

The total Hamiltonian is then given by the sum of the two separate Hamiltonians and the interaction term

$$H=H_{leads}+H_{wire}+H_{leads-wire} \tag{4}$$

The tunneling current operator of the $\gamma$th lead can be formulated by using the continuity equation and Heisenberg equation. For our system, the current operator of the left lead can be expressed as

$$\hat{I}_L(t)=\frac{ie}{\hbar}\sum_{k\sigma}[T_L c_{L,k\sigma}^\dagger(t)c_{1\sigma}(t)-T_L^* c_{1\sigma}^\dagger(t)c_{L,k\sigma}(t)] \tag{5}$$

The spectral density of shot noise is defined by the Fourier transformation of the current correlation

$$\Pi_{LL}(t,t')=\langle\delta\hat{I}_L(t)\delta\hat{I}_L(t')\rangle+\langle\delta\hat{I}_L(t')\delta\hat{I}_L(t)\rangle \tag{6}$$

where $\delta\hat{I}_\gamma(t)=\hat{I}_\gamma(t)-\langle\hat{I}_\gamma(t)\rangle$. The symbol $\langle\cdots\rangle$ in the above formula denotes the quantum expectation value of the electron state and the ensemble average over the system.

Substituting the current operator Eq. (5) into the correlation function Eq. (6), we encounter the expectation value of the current operator, and the four operator terms exhibit in the formula. We employ Wick's theorem in our system, and the ensemble average of the products of four operators are expressed by the ensemble average of the products of two operators. Then, the correlation function can be expressed by Keldysh Green's function. We then solve these Keldysh Green's function by the equation of motion (EOM) method and Langreth relations [37]. Making the Fourier transformation over the two times $t$ and $t'$, and using the relation $S_{LL}(\Omega)\delta(\Omega+\Omega')=\frac{1}{2}\Pi_{LL}(\Omega,\Omega')$, we finally obtain the zero frequency shot noise of left lead in spin$\otimes$Nambu space as follows:

$$\begin{aligned}S=-\frac{2e^2}{h}\mathrm{tr}\int d\epsilon[&\mathbf{G^r\Sigma_{11}^<G^r\Sigma_L^>}+\mathbf{G^a\Sigma_{11}^<G^a\Sigma_{11}^>}+\mathbf{G^r\Sigma_{11}^<G^>}(\mathbf{\Sigma_{11}^a}-\mathbf{\Sigma_{11}^r})\\&+\mathbf{G^<}(\mathbf{\Sigma_{11}^r}-\mathbf{\Sigma_{11}^a})\mathbf{G^>\Sigma_{11}^r}+(\mathbf{\Sigma_{11}^r}-\mathbf{\Sigma_{11}^a})\mathbf{G^<\Sigma_{11}^>G^a}\\&+\mathbf{G^<}(\mathbf{\Sigma_{11}^a}-\mathbf{\Sigma_{11}^r})\mathbf{G^r\Sigma_{11}^>}+\mathbf{G^>\Sigma_{11}^<G^a}(\mathbf{\Sigma_{11}^r}-\mathbf{\Sigma_{11}^a})\\&+\mathbf{G^<}(\mathbf{\Sigma_{11}^a}-\mathbf{\Sigma_{11}^r})\mathbf{G^>\Sigma_{11}^a}-(\mathbf{\Sigma_{11}^<G^>}+\mathbf{\Sigma_{11}^>G^<})]\end{aligned} \tag{7}$$

The current formula can be found by taking ensemble average and time average over the current operator given in Eq. (5) [38]. After some algebra calculations, we obtain the current formula of the left lead [32]

$$I_L=\frac{e}{h}\int d\epsilon Re\,\mathrm{tr}\{\boldsymbol{\sigma}[\mathbf{G^<\Sigma_{11}^a}+\mathbf{G^r\Sigma_{11}^<}]\} \tag{8}$$

where $\sigma=\mathrm{diag}(1,-1,1,-1)$ accounts for the different charges carried by the electrons and holes. $\mathbf{G^{r,a,<,>}}$ are Green's functions of nano wires in the spin $\otimes$ Nambu space. The retarded and advanced Green's functions can be written as

$$\mathbf{G^{r,a}}(\epsilon)=[\epsilon-\mathbf{H_{wire}}-\mathbf{\Sigma^{r,a}}]^{-1} \tag{9}$$

where $\mathbf{H_{wire}}$ corresponds to the matrix form of Eq. (1). $\mathbf{\Sigma^{r,a,<,>}}$ are the self-energies due to the coupling between the leads and nanowires. It has nonzero diagonal elements only at the two ends of nanowire, i.e., $\Sigma_{11}^{r,a}=\mp i/2\mathrm{diag}(\Gamma_L,\Gamma_L,\Gamma_L,\Gamma_L)$ and $\Sigma_{NN}^{r,a}=\mp i/2\,\mathrm{diag}(\Gamma_R,\Gamma_R,\Gamma_R,\Gamma_R)$. The less self-energies $\Sigma_{11}^<=i\Gamma_L\mathrm{diag}[f(\epsilon-\mu_L),f(\epsilon+\mu_L),f(\epsilon-\mu_L),f(\epsilon+\mu_L)]$ and $\Sigma_{NN}^<=i\Gamma_R\,\mathrm{diag}[f(\epsilon-\mu_R),f(\epsilon+\mu_R),f(\epsilon-\mu_R),f(\epsilon+\mu_R)]$. We can obtain $\Sigma_{11}^<$ from the relation $\Sigma_{11}^>-\Sigma_{11}^<=\Sigma_{11}^r-\Sigma_{11}^a$. Keldysh Green's functions $G^{<,>}$ can be derived by applying the Langreth relation to the Dyson equations for the retarded Green's function. Finally, we obtain

$$\mathbf{G^{<,>}}=\mathbf{G^r\Sigma^{<,>}G^a} \tag{10}$$

Substituting the obtained Green's functions and self-energies into Eqs. (7) and (8), we can obtain the shot noise and the corresponding current in our system.

## 3. Results and analysis

We now present the numerical calculations and examine feature of shot noise in superconducting wires with a periodic modulation of the chemical potential at zero temperature. We take $\mu_R$ as the