Solid State Communications 191 (2014) 10–13

Contents lists available at ScienceDirect

Solid State Communications

journal homepage: www.elsevier.com/locate/ssc

Significance of an external magnetic field on two-phonon processes in gated lateral semiconductor quantum dots

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ARTICLE INFO

Article history:
Received 17 September 2013
Received in revised form
10 April 2014
Accepted 13 April 2014
by A. Pinczuk
Available online 21 April 2014

Keywords:
A. Quantum dots
A. Semiconductors
D. Decoherence

ABSTRACT

Theoretical and numerical calculations of two-phonon processes on gated lateral semiconductor quantum dots (QDs) are outlined. A heterostructure made with two laterally coupled QDs, in the presence of an external magnetic field, has been employed in order to study the electron scattering rate due to two-phonon processes. The formalism is based on the acoustic phonon modes via the unscreened deformation potential and the piezoelectric interaction whenever the crystal lattice lacks a center of inversion symmetry. The rates are calculated by using second order perturbation theory. The strong dependence of the scattering rate on the external magnetic field, lattice temperature and QDs separation distance is presented.

1. Introduction

The gated lateral semiconductor quantum dots (QDs) in a quantum well (QW), in which the growth direction (z direction or vertical direction) confinement is due to the higher bandgap of the barrier material [1–8], have been proposed for use as quantum bits (qubits) in quantum computer architecture several times. Decoherence due to single electron confinement within a coupled QDs structure, which plays the role of qubits (QBs), has two important channels. The first channel is the Coulomb interaction to the background charge fluctuation and the second is the electron–phonon interaction. Furthermore, physical properties of semiconductor-based QBs relevant to single electron spin and charge degree of freedom have been a subject of theoretical studies [6]. Notice that for charge qubits there is only a single electron in a double dot, in contrast with spin qubits, where each quantum dot has one electron and the double dot is only for two-qubit operations [1,2].

The effects of charge decoherence due to electron–phonon interactions are of crucial importance in semiconductor-based quantum computer architecture. In our recent works [9,10], we have shown that the scattering rates due to electron–acoustical phonon interactions and the dephasing rates due to the coupling of electrons to acoustical and optical phonons strongly depend on the interdot distance and the strength of the electron confinement. The multiphonon processes [11] among other scattering mechanisms (e.g. one phonon process, electron–electron, and spin–phonon interactions) can describe and measure the decoherence in QDs. Earlier theoretical work, in multiphonon processes in single three dimensional QD made with GaAs in which the electron confinement potential is assumed to be isotropic and parabolic, was reported two decades ago [12]. In their theoretical work, they used a few possible processes using longitudinal acoustical (LA) and longitudinal optical (LO) phonons which were mainly described by bulk phonon approximation [13]. This theory serves laser nanotechnology interests and handles the photoluminescence degradation in small QDs. According to the best knowledge of the authors, the relaxation rates (source of charge decoherence in qubits and in optoelectronic devices) due to multiphonon processes in laterally coupled QDs in the presence of an external magnetic field and for a range of operating temperatures have not been reported.

In this paper, our study related to two phonon processes in coupled QDs under the existence of an external magnetic field and their role in charge decoherence is presented. Starting with Section 2, we firstly give a theoretical motivation of the electron wavefunctions and phonon model which describes the deformation and piezoelectric types of electron–phonon interactions. Secondly, we present the equation of electron scattering rates due to the second order perturbation term which describes the two-phonon processes. In Section 3, we show the relaxation rates due to two-phonon processes and we discuss their dependence on several configurations. Lastly, Section 4 presents a summary of our results and future implementations.

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http://dx.doi.org/10.1016/j.ssc.2014.04.008
0038-1098/

2. Theory

We consider a heterostructure composed of two laterally coupled QDs. In order to calculate the electron states within the coupled system, we have used a one-band effective mass approximation. The Hamiltonian which describes the single-electron motion [14] which is confined in laterally coupled QDs is given by

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_{\parallel} + \hat{\mathcal{H}}_z \tag{1}$$

where the lateral motion of electron is decoupled from the one along the quantum well growth (z-axis) [9,15]. The external magnetic field is applied along the z-axis ($\mathbf{B} = B\hat{\mathbf{e}}_{\mathbf{z}}$), and as a result the magnetic vector potential $\mathbf{A}$ could be given as

$$\mathbf{A} = B(-y\hat{\mathbf{e}}_{\mathbf{x}} + x\hat{\mathbf{e}}_{\mathbf{y}})/2 \tag{2}$$

The Hamiltonian operators for the lateral directions and z-direction have been considered as

$$\hat{\mathcal{H}}_{\parallel} = \frac{\hat{p}^2}{2m^*} + \frac{1}{2}m^*\omega^2 r_{\parallel}^2 - \frac{1}{2}\omega_c \mathbf{L}_z \tag{3}$$

$$\hat{\mathcal{H}}_z = -\frac{\hbar}{2}\partial_z \frac{1}{m^*(z)}\partial_z + V_0\Theta(|z| - L_0). \tag{4}$$

where $\mathbf{L}_z$ is the operator of the z component of the angular momentum, $m^*(z)$ is the electron effective mass, V_0 is the offset between the band edges of the GaAs well and the AlGaAs barrier, Θ is the Heaviside step function, $\hat{p}$ is the quantum mechanical operator of momentum, ω_0 is a parameter describing the strength of the confinement in the x–y plane, $\omega = Be/m^*$ and $\omega^2 = \omega_0^2 + (\omega_c/2)^2$.

According to Eq. (1), electron wavefunction can be given by the following envelope function:

$$\psi(\mathbf{r}) = \psi_{\parallel}(\mathbf{r}_{\parallel})\psi_z(z). \tag{5}$$

In our investigation, we have only considered the ground state wavefunction along the QW growth and the wavefunction along the lateral direction is given by Fock–Darwin states. Following the same procedure as [9], we have considered that the external confining potential for the electron within two QDs structure is given by

$$V_c = \tfrac{1}{2}m^*\omega_0^2 \min\left\{(x-\alpha)^2 + y^2, (x+\alpha)^2 + y^2\right\} \tag{6}$$

where α is the separation distance of the dots. The electron wavefunction of the coupled QD structure could be described by

$$\Psi(\mathbf{r}) = \Psi_{\parallel}(\mathbf{r}_{\parallel})\psi_z(z) \tag{7}$$

where the single electron wavefunction for the parallel plane is given by

$$|\Psi_{\parallel}\rangle = \sum_k C_k|\psi^k_{\parallel,L}\rangle + D_k|\psi^k_{\parallel,R}\rangle. \tag{8}$$

A numerical scheme has been employed in order to calculate the total wavefunction in the parallel plane of the coupled dot system. In low dimensional structures, the electrons interact with acoustical and optical phonons. The optical phonons do not have any contribution to electron scattering rates due to the small electron energy splitting. Therefore, only the acoustical phonons contribute to the relaxation rates. In this work, we calculate the electron scattering rate which is caused due to deformation potential and piezoelectric acoustic phonon interaction [16–18]. The Hamiltonian which describes these interactions is given by

$$H = \sum_{\mathbf{q}}\left(\frac{\hbar}{2\rho_m V\omega_{\mathbf{q}}}\right)^{1/2}\mathcal{M}(\mathbf{q})\rho(\mathbf{q})(a_{\mathbf{q}} + a^{\dagger}_{-\mathbf{q}}). \tag{9}$$

The term $\mathcal{M}(\mathbf{q})$, which includes both the deformation and the piezoelectric interaction for zincblende crystals, is defined by

$$\mathcal{M}(\mathbf{q}) = D|\mathbf{q}| + i\mathcal{M}^{pz}_{\lambda}(\hat{q}) \tag{10}$$

with

$$\mathcal{M}^{pz}_{\lambda}(\hat{\mathbf{q}}) = 2e\ e_{14}(\hat{q}_x\hat{q}_y\xi_z + \hat{q}_y\hat{q}_z\xi_x + \hat{q}_x\hat{q}_z\xi_y). \tag{11}$$

In Eqs. (9)–(11), ρ_m is the mass density of the host material, $\omega_{\mathbf{q}}$ is the frequency of the phonon mode with wavevector $\mathbf{q}$, V is the volume of the sample, $a_{\mathbf{q}}$ and $a^{\dagger}_{-\mathbf{q}}$ are phonon annihilation and creation operators, $\rho(\mathbf{q})$ is the electron density operator, D denotes the deformation potential, e_{14} is the piezoelectric constant and ξ is the polarization vector. All values of the abovementioned parameters used in our calculations have been taken from Ref. [18].

The last part of our theoretical formalism is the calculation of the electron scattering rates due to two-phonon processes. Considering only LA phonons, the scattering rates (second order perturbation theory) are given by the following equations:

$$\Gamma_{++} = \frac{\pi}{\hbar}\sum_{\mathbf{q}}\sum_{\mathbf{k}}\left|\sum_s\left(\frac{M^{is}_q M^{sf}_k}{E_i - E_s - E_q} + \frac{M^{is}_k M^{sf}_q}{E_i - E_s - E_k}\right)\right|^2 \times (N_q + 1)(N_k + 1)\delta(E_i - E_f - E_q - E_k) \tag{12}$$

$$\Gamma_{+-} = \frac{2\pi}{\hbar}\sum_{\mathbf{q}}\sum_{\mathbf{k}}\left|\sum_s\left(\frac{M^{is}_q M^{sf}_k}{E_i - E_s - E_q} + \frac{M^{is}_k M^{sf}_q}{E_i - E_s + E_k}\right)\right|^2 \times N_k(N_q + 1)\delta(E_f - E_i - E_q + E_k) \tag{13}$$

$$\Gamma_{--} = \frac{\pi}{\hbar}\sum_{\mathbf{q}}\sum_{\mathbf{k}}\left|\sum_s\left(\frac{M^{is}_q M^{sf}_k}{E_i - E_s + E_q} + \frac{M^{is}_k M^{sf}_q}{E_i - E_s + E_k}\right)\right|^2 \times N_q N_k\delta(E_f - E_i + E_q + E_k) \tag{14}$$

where the indices $++$, $--$, $+-$ represent the emission of two phonons (LA+LA), the absorption of two phonons ($-$LA$-$LA) and the emission of one phonon and absorption of one phonon (LA$-$LA or $-$LA+LA) respectively. M^{sf}_q stands for the electron–phonon matrix elements where the index i (f) corresponds to the qubit electron first excited state (ground state) and s stands for the intermediate electronic states. The other elements are taken by changing the proper suffices. N_k (N_q) is the Bose distribution function referring to phonons with energy $E_k = \hbar\omega_k$ ($E_q = \hbar\omega_q$). Note that the summation over s excludes the initial and final states. The integrals, which are included in Eqs. (12)–(14) by transforming the summations to integrations, have been calculated by the Monte Carlo code.

3. Results

Fig. 1 shows all possible scattering processes concerning the electron transitions due to second order contributions associated with acoustic phonons. The transitions described by Eqs. (12) and (14) are presented in Fig. 1-I (Fig. 1b) and Fig. 1III (Fig. 1c) respectively. It is worth mentioning that Eq. (13) creates two different transitions as illustrated in Fig. 1-II (Fig. 1b) and Fig. 1-IV (Fig. 1d). Using the results of the second order perturbation theory (Eqs. (12)–(14)), we estimate the relaxation rates for an electron which relaxes to the ground state via the two phonon processes.

In Fig. 2, we present the relaxation rates for the case of the emission of a LA phonon and the absorption of a LA phonon (LA$-$LA), as a function of an external magnetic field. Increasing the magnetic field in the range of 0–12 T, the electron wavefunctions get the largest value (resonance value) at B 3.7 T and as a result the matrix elements involved in the two-phonon scattering process increase. Furthermore, the increasing number of phonon modes (for B=0–4 T) that can be involved in the relaxation