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Asymmetric field dependence of specific heat in two-leg Heisenberg antiferromagnetic spin ladders with asymmetric leg interactions

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ABSTRACT

Two-leg Heisenberg antiferromagnetic spin-1/2 ladders with asymmetric leg interactions ($J_1 \neq J_2$) are studied using the bond-mean field method. With different $J_1-J_2-J_\perp$ couplings, magnetization process exhibits distinct properties. For J_1:J_2:$J_\perp$=4:2:1 and 4:1:2, M–h curves exhibit cusp-type singularities which do not appear for J_1:J_2:$J_\perp$=2:1:4. Field dependence of specific heat (C_v/T–h curve) has been found to be asymmetric about $h_m=(h_1+h_2)/2$, h_1 and h_2 are two critical fields. The cusp in M–h curves and the asymmetric structure of C_v/T–h curves are closely related to their energy spectrums. The linearity of specific heat in the gapless Luttinger-liquid phase and the underlying asymmetric ladder structure can be characterized by C_v/T–h curves.

1. Introduction

Spin ladders, which are composed of exchange-coupled $S=1/2$ one-dimensional (1D) Heisenberg antiferromagnetic (HAF) chains, have been in considerable interest as an ideal model for revealing the transitional behavior from one- to two-dimensional systems [1–3]. Much effort has been made to study spin ladders [4–11]. Spin 1/2 HAF ladders with even legs have a spin gap and short-range correlations, while ladders with odd legs have no spin gap and power-law correlations [5]. This has been verified both experimentally and theoretically [4–6,12–15]. Recently, the symmetric behavior of the field dependence of specific heat has been found [8–11]. On the other hand, the dimerized spin ladders have also been investigated widely due to the novel low-lying excitations [16–21]. Thus far, little work has been carried out on spin ladders with asymmetric legs. Only the zigzag ladders with asymmetric legs have been explored to investigate the effect of geometric frustration in the low-dimensional spin systems [22–27]. Using bosonization and renormalization group approaches, it has been shown that an asymmetric zigzag spin ladder lies in different ground states when the leg exchange integrals change [22,23]. Studies on two-leg spin ladders with unequal legs are little, due to the fact that asymmetric spin ladders have not been realized experimentally.

Herein, we propose a theoretical model for the two-leg asymmetric ladders and have investigated their magnetic and thermodynamic properties. We aim to determine the influence of the asymmetric ladder structures on the magnetization and thermodynamics. This paper is organized as follows. The model Hamiltonian and the method we used are given in Section 2. In Section 3, the magnetization of asymmetric ladders are calculated and discussed. Also, the field dependence of specific heat is presented. Finally, in Section 4, a brief summary will be given.

2. Model and method

The Hamiltonian of an asymmetric spin ladder (see Fig. 1) we studied is given by

$$H=J_1\sum_{i}^{i=N} S_{i,1}S_{i+1,1}+J_2\sum_{i}^{i=N} S_{i,2}S_{i+1,2}+J_\perp\sum_{i}^{i=N} S_{i,1}S_{i,2}-h\sum_{i}^{i=N}\sum_{j=1}^{2} S^z_{i,j} \tag{1}$$

where $h=g\mu_B B$ is the Zeeman energy term, j is a chain index, i is the rung index, J_1 and J_2 are HAF exchange coupling integrals along the two chains, $J_1 \neq J_2$. $J_\perp$ is the HAF exchange couplings integral along the rung. N is the total number of rungs, and periodic boundary condition along the chain direction is assumed.

After the Jordan–Wigner (JW) transformation,

$$S^+_{i,j}=c^+_{i,j}e^{i\phi_{i,j}}$$

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$$\phi_{i,1}=\pi\left[\sum_{d=0}^{i-1}\sum_{f=1}^{2} n_{d,f}\right] \quad \text{for chain 1,}$$

$$\phi_{i,2}=\pi\left[\sum_{d=0}^{i-1}\sum_{f=1}^{2} n_{d,f}+n_{i,1}\right] \quad \text{for chain 2,} \tag{2}$$

$$S_{i,j}^{z}=n_{i,j}-1/2,$$

$c_{i,j}^{+}(c_{i,j})$ is the spinless fermion creation (annihilation) operator, $n_{i,j}=c_{i,j}^{+}c_{i,j}$ is the occupation number operator, then Eq. (1) becomes

$$\begin{aligned}H=&\sum_i \frac{J_1}{2}e^{i(\phi_{i,1}-\phi_{i+1,1})}c_{i,1}^{+}c_{i+1,1}+\sum_i \frac{J_2}{2}e^{i(\phi_{i,2}-\phi_{i+1,2})}c_{i,1}^{+}c_{i+1,1}\\&+\sum_i \frac{J_\perp}{2}e^{i(\phi_{i,1}-\phi_{i,2})}c_{i,1}^{+}c_{i,2}+(H.c.)+\sum_i J_1\left(n_{i+1,1}-\frac{1}{2}\right)\left(n_{i+1,1}-\frac{1}{2}\right)\\&+\sum_i J_2\left(n_{i+1,2}-\frac{1}{2}\right)\left(n_{i+1,2}-\frac{1}{2}\right)+J_\perp\sum_i\left(n_{i,1}-\frac{1}{2}\right)\left(n_{i,2}-\frac{1}{2}\right)\\&-h\sum_i\sum_{j=1}^{2}\left(n_{i,j}-\frac{1}{2}\right)\end{aligned} \tag{3}$$

This describes a system of interacting spinless JW fermions in an external field. Then, the bond-mean field method (BMFT) is used to treat the phases $\phi_{i,j}$ [19,28–30]. The quartic Ising terms in Eq. (3) are treated within the Hartree–Fock approximation as follows:

$$\begin{aligned}&\left(n_{i,j}-\frac{1}{2}\right)\left(n_{i+1,j}-\frac{1}{2}\right)\\&=\langle c_{i,j}c_{i+1,j}^{+}\rangle c_{i,j}^{+}c_{i+1,j}+\langle c_{i+1,j}c_{i,j}^{+}\rangle c_{i+1,j}^{+}c_{i,j}+\langle c_{i,j}c_{i+1,j}^{+}\rangle\langle c_{i+1,j}c_{i,j}^{+}\rangle\\&\quad+\langle c_{i,j}^{+}c_{i,j}\rangle c_{i+1,j}^{+}c_{i+1,j}+\langle c_{i+1,j}^{+}c_{i+1,j}\rangle c_{i,j}^{+}c_{i,j}-\langle c_{i,j}^{+}c_{i,j}\rangle\langle c_{i+1,j}^{+}c_{i+1,j}\rangle\\&\quad-\frac{1}{2}c_{i,j}^{+}c_{i,j}-\frac{1}{2}c_{i+1,j}^{+}c_{i+1,j}+\frac{1}{4}\end{aligned} \tag{4}$$

$Q_1=\langle c_{i,1}c_{i+1,1}^{+}\rangle$ and $Q_2=\langle c_{i,2}c_{i+1,2}^{+}\rangle$ on the two legs and $P_1=\langle c_{i,1}c_{i,2}^{+}\rangle$ along the rung, $i=1,2,3,\ldots N/2$. Further details on the BMFT can be found elsewhere [19,28–34].

In the following numerical calculations, we take J_1 as the energy unit. For simplification, we only consider the three cases below: (a) $J_1>J_2>J_\perp$, J_1:J_2:$J_\perp$=4:2:1; (b) $J_1>J_\perp>J_2$, J_1:J_2:$J_\perp$=4:1:2; and (c) $J_\perp>J_1>J_2$, J_1:J_2:$J_\perp$=2:1:4.

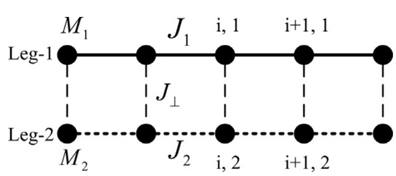

Fig. 1. Two-leg spin-1/2 ladder with asymmetric legs. J_1, J_2 are the antiferromagnetic Heisenberg exchange integrals along leg-1 and leg-2. $J_\perp$ is the antiferromagnetic Heisenberg exchange integral along the rung.

3. Results and discussions

Fig. 2 presents the magnetization M per site as a function of h for the three kinds of asymmetric ladders at absolute zero temperature. In the first case (J_1:J_2:$J_\perp$=4:2:1 in Fig. 2a), we can find that there are four distinct regions: the 0-magnetization plateau phase, the first gapless Luttinger-liquid phase, the second gapless Luttinger-liquid phase and the saturated magnetization plateau phase. This can also be confirmed from the singularities of the susceptibility. Although the exchange integrals of two legs are unequal, the gapped ground state is not affected and corresponds to $M=0$. When the external field is applied, the gapped ground state with $M=0$ still persists for $h<h_1$ because the magnetic field energy is not large enough to excite the ground state. For $h>h_1$, the plateau state is destructed and M increases as h increases. Magnetization per site on leg-(1) M_1 increases slowly to the saturated value at $h=h_2$, while magnetization per site on leg-(2) M_2 increases rapidly to the saturated value at $h=h_{cusp}$ and then

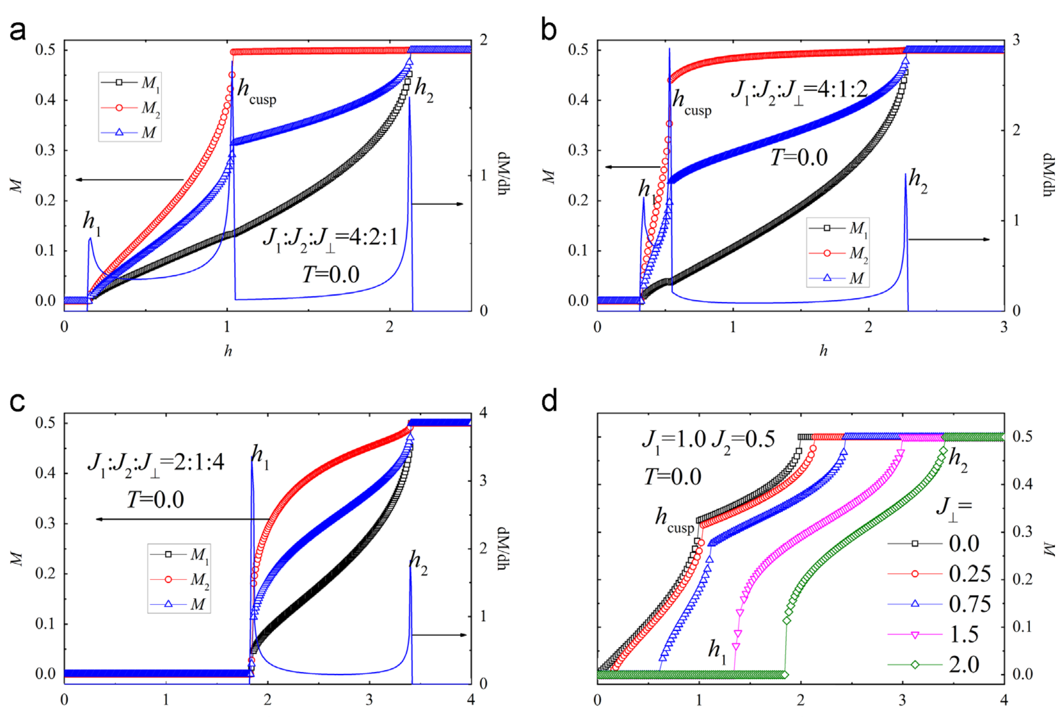

Fig. 2. (Color online) Field dependence of magnetization for the three cases: (a) $J_1>J_2>J_\perp$, J_1:J_2:$J_\perp$=4:2:1; (b) $J_1>J_\perp>J_2$, J_1:J_2:$J_\perp$=4:1:2; (c) $J_\perp>J_1>J_2$, J_1:J_2:$J_\perp$=2:1:4; (d) $J_1=1.0$, $J_2=0.5$, $J_\perp=0.0$, 0.25, 0.75, 1.5 and 2.0. M_1 and M_2 are the magnetization per site on the different legs: leg-(1) and leg-(2). The singularities of the susceptibility as a function of h are plotted, which indicate the occurrence of quantum phase transitions. $T=0.0$.