Solid State Communications 178 (2014) 23–27

Contents lists available at ScienceDirect

Solid State Communications

journal homepage: www.elsevier.com/locate/ssc

Phase diagram of microcavity polariton condensates with a harmonic potential trap

Ting-Wei Chen [a], Ming-Dar Wei [a], Szu-Cheng Cheng [*,b], Wen-Feng Hsieh [*,c]

[a] Department of Photonics, National Cheng Kung University, Tainan, Taiwan
[b] Department of Optoelectric Physics, Chinese Culture University, Taipei, Taiwan
[c] Department of Photonics and Institute of Electro-Optical Engineering, National Chiao Tung University, Hsinchu, Taiwan

ARTICLE INFO

Article history:
Received 12 July 2013
Accepted 30 September 2013
by Y.E. Lozovik
Available online 24 October 2013

Keywords:
D. Polariton condensates
D. Quantized vortices

ABSTRACT

We theoretically explore the phase transition in inhomogeneous exciton-polariton condensates with variable pumping conditions. Through Bogoliubov excitations to the radial-symmetric solutions of complex Gross–Pitaevskii equation, we determine not only the bifurcation of stable and unstable modes by the sign of fluid compressibility but also two distinct stable modes which are characterized by the elementary excitations and the stability of singly quantized vortex. One state is the quasi-condensate BKT phase with Goldstone flat dispersion; the other state is the localized-BEC phase which exhibits linear-type dispersion and has an excitation energy gap at zero momentum.

1. Introduction

It is known that there is no Bose–Einstein condensate (BEC) in the thermodynamic limit at any finite temperature in homogeneous two-dimensional (2D) systems, and instead superfluidity can be achieved through Berezinskii–Kosterlitz–Thouless (BKT) transition in an interacting system. The emergence of superfluidity was interpreted by the formation of a topological order resulting from vortex–antivortex pairs (VAPs) [1–3]. The BKT theory for a uniform system predicts a density jump of superfluid at the transition [4], namely, the transition happens at a universal ratio of $n_s \lambda_T^2 = 4$, where n_s is the superfluid density and λ_T is the thermal de Broglie wavelength of BEC. However, the universality for 2D trapped systems was experimentally observed to be 6 ± 2 in cold atoms [5].

In parallel to the pursuing of BEC in semiconductor microcavities [6–9], microcavity polariton condensate (MPC) was proposed as a candidate to study superfluid, which is particularly characterized by the formation of vortices [10–12], collective dynamics as well as elementary excitations with linear dispersion [13]. In analogy to the trapped 2D cold atoms, the major difference of MPCs is the local self-equilibrium property coming from the finite lifetime of exciton polarions. Inhomogeneous 2D polariton BECs with a continuous replenishment have been theoretically studied in use of the modified complex Gross–Pitaevskii equations (cGPEs) [14]. The thermodynamic properties of 2D exciton-polaritons had been analyzed and shown to exhibit local condensation or BKT phase transition towards superfluidity [15]. In addition, the dynamical evolution of the condensate was investigated by a quantum kinetic formalism with the distribution function of polaritons described by a semi-classical Boltzmann equation and phase diagrams for several microcavities were obtained with respect to the temperature and polariton density [15]. Similar phase diagram had been calculated with randomly distributed disorders based on cGPEs, in which quasi-particle excitation spectra were also discussed for different phases [16]. Furthermore, the BKT-like phase was investigated that used spatial correlation to observe the power-law decay indicating the existence of BKT-like phase in open-dissipative systems [17].

Instead of considering the random disorders, we show in this paper by studying quasi-particle excitations to the cGPE with a harmonic trap assuming the relatively slow dynamics of the condensate by eliminating the reservoir effect so that the dynamics can be easily studied by a single partial differential equation. In use of the cGPE accompanied with Bogoliubov excitations for studying the 2D polariton condensates in a harmonic trap, a bifurcation of stable and unstable modes can be obtained associated with the sign of fluid compressibility. Apart from the criterion for instability of the vortex-free state, we conclude a 2D trapped polariton condensate may exhibit both the BEC and BKT-like phases, separated by what we believe to be a quasi-condensation boundary. The phase diagram is established through the pumping spot and strength rather than the temperature and polariton density in those articles aforementioned.

* Corresponding authors at: Department of Optoelectric Physics, Chinese Culture University, Taipei, Taiwan. Tel.:+886 228610511. Department of Photonics and Institute of Electro-Optical Engineering, National Chiao Tung University, Hsinchu, Taiwan. Tel.: +886 35712121.
E-mail addresses: sccheng@faculty.pccu.edu.tw (S.-C. Cheng), wfshieh@mail.nctu.edu.tw (W.-F. Hsieh).

0038-1098/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ssc.2013.09.035

2. Condensate wave functions and elementary excitations

The GPE is a mean-field model to deal with the many-body problems [18,19]. The classical field ψ, known as the macroscopic wavefunction of the polariton condensate, is used to replace the field operator by ignoring the non-condensate part from quantum and thermal fluctuations. Under the assumption that the polariton gas is weakly-interacting, the condensate dynamics can be described by the cGPE without microscopic physics of the polaritons involved. Here, a theoretical model is presented for a 2D finite system of MPCs as

$$i\hbar\frac{\partial\psi}{\partial t}=\left(-\frac{\hbar^2}{2m}\nabla^2+V_{ext}+U|\psi|^2\right)\psi+i(\gamma_{eff}-\Gamma|\psi|^2)\psi, \quad (1)$$

where the external trapping potential is given by the harmonic form of $V_{ext}=m\omega^2r^2/2$, with ω being the angular frequency, m the polariton mass, r the radial coordinate. U is the interaction energy similar to the optical Kerr constant; γ_{eff} is the linear net gain describing the balance of the stimulated scattering of polaritons into the condensate and the decay of polaritons out of the cavity. This is actually a spatially dependent gain determined by the pumping scheme and Γ represents the coefficient of nonlinear scattering loss. We assume the pump profile has a form of polar symmetric step-function with amplitude γ_{eff} and pump spot size R.

Let the oscillator length be $L=\sqrt{\hbar/m\omega}$, so that the length, time and energy are in units of L, $1/\omega$ and $\hbar\omega$, respectively. Under Thomos-Fermi scaling, we rescale ψ_k to be $\sqrt{\hbar\omega/2U}\Phi$ and introduce the dimensionless variables $\alpha=2\gamma_{eff}/\hbar\omega$ and $\sigma=\Gamma/U$ to represent the pumping strength and scattering loss. The mean-field cGPE can be written as

$$i\frac{\partial\Phi}{\partial t}=\left[-\frac{1}{2}\nabla^2_{\vec{\rho}}+\frac{\rho^2}{2}+\frac{1}{2}|\Phi|^2+\frac{i}{2}(\alpha-\sigma|\Phi|^2)\right]\Phi. \quad (2)$$

here the Laplacian operator $\nabla^2_{\vec{\rho}}$ is associated with the dimensionless polar coordinate $\vec{\rho}=(\rho,\theta)$ with $\rho=|\vec{r}|/L$. With σ fixed throughout this paper, the nature of the system can be quantified by two control parameters, i.e., α and R, which are both accessible experimentally. The chemical potential $\mu(T)$ at quasi-thermal equilibrium temperature T is related to the radially symmetric wavefunction by $\Phi(\rho,t)=\Phi(\rho)e^{-i\mu(T)t}$, and it will be determined once the pumping condition is specified. Therefore, the cGPE can be written as

$$\left[-\frac{1}{2}\nabla^2_{\vec{\rho}}+\frac{\rho^2}{2}+\frac{1}{2}|\Phi|^2+\frac{i}{2}(\alpha-\sigma|\Phi|^2)\right]\Phi=\mu(T)\Phi. \quad (3)$$

The steady-state nonlinear GPE is numerically solved in use of the Runge–Kutta evolution and the shooting method with proper boundary conditions and additional constraint to enforce the population conservation of pumping and loss. The chemical potential for a given pumping scheme is targeted to fulfill both the boundary conditions and the conservation law of polaritons $\int(\alpha-\sigma\Phi^2)|\Phi|^2d^2r=0$, i.e., the balance between net gain and loss over all the space should be zero.

Once the stationary solutions are obtained, the stability can be investigated by means of Bogoliubov-de Genned analysis. We consider a small deviation $\delta\Phi=e^{-i\mu t}[w(\rho,\theta)e^{-i\Omega t}-v^*(\rho,\theta)e^{i\Omega t}]$ from the radially symmetric stationary solution $\Phi_0(\rho)=A(\rho)e^{i\phi(\rho)}e^{-i\mu t}$, where $w(\rho,\theta)=W(\rho,\theta)e^{i\vec{k}\,\vec{\rho}}e^{i\phi(\rho)}$ and $v(\rho,\theta)=V(\rho,\theta)e^{i\vec{k}\,\vec{\rho}}e^{i\phi(\rho)}$; then we substitute the total wavefunction $\Phi=\Phi_0+\delta\Phi$ into Eq. (3). The response of the polariton condensate under a small perturbation is calculated by linearizing the cGPE around the stationary solution in the momentum space. We assume the fluctuation is of an angular hormonic function multiplied by a radial Bessel function such that $W(\rho,\theta)=\sum_{m,n}(A_{mn}/2\pi)e^{im\theta}J_m(k_{mn}\rho)$ and $V(\rho,\theta)=\sum_{m,n}(B_{mn}/2\pi)e^{im\theta}J_m(k_{mn}\rho)$ where k_{mn} multiplied by the size of the system R_0, i.e., $k_{mn}R_0$, represents the nth zero root of the mth-order Bessel function $J_m(\rho)$. The Bogoliubov equations derived this

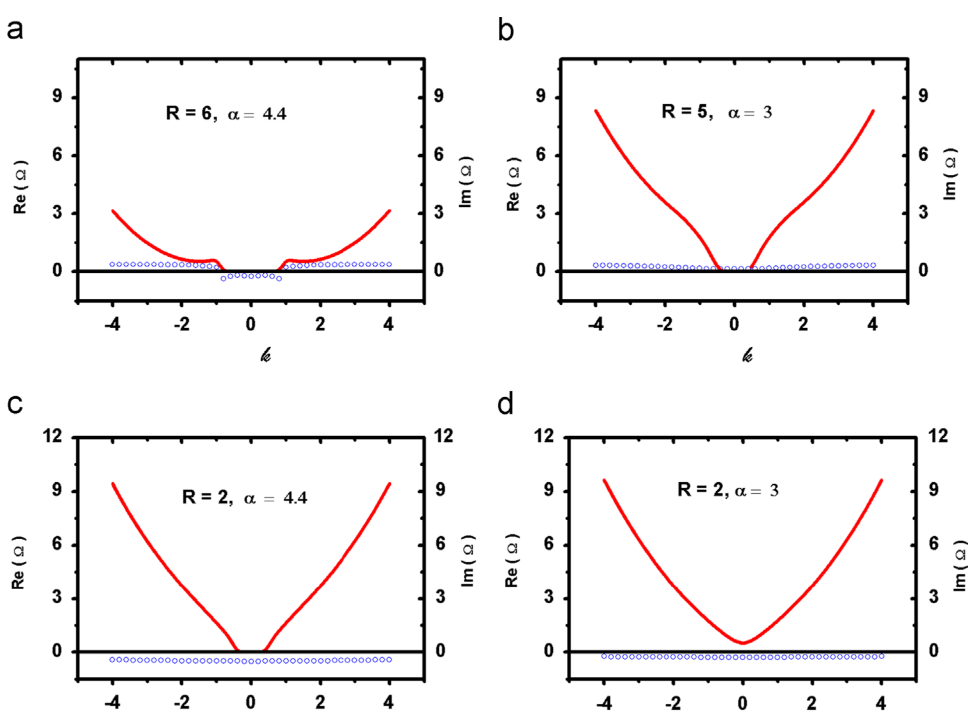

Fig. 1. (Color online) Bogoliubov excitation spectra of stable and unstable modes. (a) Unstable mode for R=6 and α=4.4, (b) unstable mode for R=5 and α=3, (c) soft mode for R=2 and α=4.4, and (d) rigid mode for R=2 and α=3. Red lines indicate the real part and blue lines indicate the imaginary part of the eigenfrequency.