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Measurement of eddy diffusivity in bubble column and validation based on the intermittency models

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Abstract

This paper discusses methods for the estimation of eddy diffusivity using the instantaneous velocity–time data measured in a bubble column reactor. In the first method, the analysis uses the eddy isolation methodology with a correction for the bubble–beam path interruption in the data. The correction is estimated from the observation of the time-varying data acquisition rate in the time series. The method is oblivious to the type of anemometer used for the data acquisition and is useful for all kinds of multiphase measurements. For validation of the results, we have proposed a strategy based on the synergistic combination of energy spectrum and the intermittency models for revealing different stages in the turbulent cascades. The method uses the actual scales in the cascade for the estimation of eddy diffusivity and hence such a combination has resulted in a robust validation tool. The comparison of the estimations based on the standard k–ε model and integral length scales is also discussed.

Keywords: Bubble column reactor; Bubbly flow; Eddy diffusivity; LDA; Time series; Intermittency

1. Introduction

Turbulent viscosity or eddy diffusivity (v_t) is a very old concept. Its origin can be found in Maxwell's work on the kinetic theory, later analysed by Saint-Venant (1851) and then extended by Boussinesq (1870), Taylor (1915) and Prandtl (1925). According to the kinetic theory of gases, the molecular viscosity of a fluid is proportional to the product of the molecular mean free path and the average speed of the molecules. By analogy, the eddy diffusivity can also be expressed as a product of the characteristic turbulence length scales and velocity scales. In the initial stages of the development of this concept (viz. mixing length theory) it was defined in terms of the integral scales and the mean flow parameters. In reality, the eddy diffusivity is used for turbulence modelling and hence it becomes hard to justify the use of eddy diffusivity based on mean flow parameters. Six decades ago, Chou (1945) had developed the basis of the approach for providing the turbulence closure based on the equations for the moments of turbulence quantities using the two-point correlation technique. The use of various turbulence models (discussed in the next paragraph) has also become popular over the last three decades. As is well known, turbulence is a compendium of motions over several scales (Sreenivasan, 1999), and hence defining the eddy diffusivity on the basis of moments of velocity and the integral parameters seems to be a jargon. Thus, it is required to understand whether the turbulent viscosity also needs a definition in terms of turbulent parameters over different scales and then validate the results using the experimentally measured eddy diffusivity using a systematic methodology or an altogether different approach is needed based on the known turbulence quantities. The present paper focuses on the first issue. Before clearly illustrating the objective of this work, here we briefly discuss the importance of the concept and the present status of understanding about the concept of eddy diffusivity and different methods for its realization.

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Eddy diffusivity is the means through which the time and length scale effects of turbulent flows are introduced into the equations of the mean flow. Thus, analogous to the kinetic theory, modelling v_t requires specification about local length and time scales (or equivalently local velocity and length scales). In the numerical simulations of a flow field, turbulence models are used for the determination of Reynolds stresses. Typical one-point turbulence models include eddy viscosity models (which assume that the Reynolds stresses are a local property of the mean flow and are related to the mean flow gradients via turbulent viscosity; Launder and Spalding, 1974) and Reynolds stress models. These models are based on the Boussinesq assumption (1870) that the turbulent stress tensor can be expressed in terms of the mean rate of strain in the same way as the viscous stress for Newtonian isotropic fluid, except that the coefficient of the molecular viscosity is replaced by eddy viscosity ($v_t \sim$ turbulent shear stress/velocity gradient) provided the ratio on the right-hand side exists. These models are derived largely based on intuition and have an empirical form. For the details of different models and their applicability in different flow systems, the reader may refer to Rodi (1995).

The most appreciated eddy viscosity model is the k–ε model based on the dimensional analysis by Launder and Spalding (1974). The k–ε models provide the velocity scale via the modelled turbulent kinetic energy (k) and the length scale via a combination of k and the energy dissipation rate (ε) as $v_t = C_\mu \mathrm{k}^2/\varepsilon$, where $C_\mu = 0.09$. The standard k–ε model is valid only for fully turbulent flows. The major advantage of k–ε models is the simplicity and suitability for an easy incorporation into the numerical codes for CFD simulations. It is unsuitable for some irregular cases (e.g. non-equilibrium, fast evolving, separating, buoyant flows, stress-driven secondary flows in non-circular ducts) and the predictions deviate significantly. For the case of bubbly flows this problem can be overcome to some extent by following the approach by Sato and Sekoguchi (1975). In such a case, it will be desirable to check the validity of the simulated eddy diffusivity through experimentally measured values much before predicting the flow pattern.

It is important that the eddy diffusivity measurement method should be non-empirical and it should use the actual turbulence parameters. Recently, Cerutti et al. (2000) have experimentally measured the spectral eddy viscosity for a subgrid-scale dissipation spectrum using the data taken using X-wire probes behind a cylinder wake. Their experimental measurements are seen to support the predictions in a subgrid-scale LES of the high *Re* flow based on classical two-point closures. Importantly, the spectral eddy viscosity is (i) a function of the cut-off wavenumber, (ii) it is different from the actual eddy viscosity and (iii) can be expressed in various forms (Metais and Lesieur, 1992). However, no experimental attempts are seen to establish measurement method for the eddy diffusivity and suitable closures based on cascade dynamics, specifically in a multiphase flow. Here we have made an attempt to illustrate two methods for the experimental measurement of eddy diffusivity using measured instantaneous velocity–time data in a bubble column reactor. The first one is specifically for the multiphase flows, while the second one, based on the intermittency models, can be used for any flow system. In view of the proper understanding of the second methodology, here we prefer to present a summary of the different intermittency models in a qualitative manner (pertinent references have been cited, where readers may refer to the original work for quantitative perspectives).

1.1. Summary of intermittency models

Intermittency is a natural phenomenon arising out of non-equilibrated interaction of eddies with the surrounding. Towards the finer scales, the intermittency grows rapidly and non-linearly. Intermittency is also characterized by the deviation from the Gaussian probability distribution of the velocity structure functions and generally it is referred to the irregular nature of energy dissipation in an energy cascade.

The complexity in a turbulent flow field is mainly due to its sensitivity towards the various events happening over a range of wavenumbers/scales. The small scales include the dissipative range responsible for most of the energy dissipation and some tail portion of the energy transfer range (inertial range in case of isotropic turbulence). The transfer range scales are large compared to dissipative scales but small compared to the large scales that extract energy from the mean flow. For a reasonably good understanding of the subject and the various aspects of turbulence, readers may refer to Monin and Yaglom (1971), Frisch (1994) and Sreenivasan and Stolovitzky (1995). The quantitative analysis of the turbulent flows was carried out for the first time by Kolmogorov (1941a,b) through his theory of isotropic turbulence, which was later critically observed and verified by several investigators over the last few decades. Here, some of the theories (although there is a big list, we have discussed only a few which we have used) specifically pertaining to the eddy break-up (intermittency models) are summarized to create a qualitative background for the analysis part of this paper. These models can be broadly classified as models based on the original homogeneous fractal case (Kolmogorov, 1941a,b; Frisch et al., 1978), multifractal formalism (Mandelbrot, 1974; Benzi et al., 1984; Meneveau and Sreenivasan, 1987; She and Leveque, 1994) and degenerate models (Gurvich and Yaglom, 1967). Additionally, some other approaches are present in the literature based on the structure function properties and the renormalization group method. For a possible complete list of these models, the reader may refer to Monin and Yaglom (1971), Borgas (1992) and Frisch (1994). Since most of the intermittency models have some intrinsic basis from the pioneering theory of isotropic turbulence by Kolmogorov (1941a), we begin with a short description of this theory followed by subsequent contributions by other investigators that have brought the concept of intermittency to a better level.