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Film flow down a fibre at moderate flow rates

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Abstract

We consider axisymmetric fluid flow down the exterior of a rigid straight vertical cylinder and model the emergence and propagation of the finite-amplitude waves that are created at the free surface. A system of evolution equations for the film thickness and volumetric flow rate, that was first derived by Trifonov [1992. Steady-state travelling waves on the surface of a viscous liquid film falling down on vertical wires and tubes. A.I.Ch.E. Journal 38, 821–834] has been re-formulated as an extension of the classical falling film problem. To inspire confidence in the predictions of this model, its linear stability characteristics versus those of the full Navier–Stokes equations are examined yielding very good agreement. Travelling wave solutions are determined and analysed in detail over a wide range of system parameters. The solutions resemble those associated with film for sufficiently small film thickness to fibre radius ratios, and beads when these ratios are relatively large. Transient computations are also performed for comparison with the travelling wave solutions and demonstrate the selection mechanism leading to the development of so-called 'dominating' waves for comparison with experimental observations.

Keywords: Film flow; Waves; Bifurcations

1. Introduction

The flow of a thin liquid film down the outside of a vertically aligned cylinder due to gravity is of central importance to numerous industrial applications, for instance, in coating flows. Studies of this flow began with the experimental work of Kapitza and Kapitza (1949), who carried out an investigation of, the now classical, falling film problem (the flow of a thin fluid film down a vertical *flat* plane) often also called the Kapitza problem, by actually doing experiments using flow down a cylinder; the film thickness was much less than the radius of the cylinder in Kapitza's experiments. In fact, a large number of the experimental studies for falling films are actually conducted using flow down cylinders due to simplifications in the measuring techniques afforded by this setting. Although it is similar in many respects to the falling film problem, the flow of a thin film on the outside of a cylinder has an important distinguishing feature: the azimuthal curvature of the interface can, for sufficiently large film thickness to cylinder radius ratio, give rise to a Rayleigh instability, which, in the case of jets or threads, can lead to breakup. Thus, surface tension can play both a stabilising *and* a destabilising role due to the axial and azimuthal film curvatures, respectively.

Modelling work on the flow of thin films down the outside of cylinders began with the work of Goren (1962) who analysed the linear stability of an annular liquid thread at rest for different cases depending on the values of two parameters, $\sigma R/\rho \nu^2$ and $R/(R+H)$; here, ρ, ν and σ are the density, the kinematic viscosity and the surface tension, respectively, and R and H are the radius of the solid substrate and the film thickness, respectively. Goren found his results to be in good agreement with experimental data. A detailed linear stability analysis of this flow was carried out by Solorio and Sen (1987) using both asymptotic and numerical methods. These authors showed that increasing the azimuthal curvature of the film is destabilising, as expected, leading to an increase in the range of unstable wavenumbers.

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A number of stability analyses were also carried out on the flow starting from nonlinear equations derived using asymptotic methods. For instance, a linear stability analysis was also carried out by Lin and Liu (1975) within the framework of nonlinear equations derived from the full Navier–Stokes equation for small-amplitude long waves. It was shown there that the flow becomes unstable as the ratio H/R increases and that the cut-off wavelength as $U_s H/\nu \to 0$ is $\tilde{L}_z = 2\pi H(1+\varepsilon)/\varepsilon$; here, U_s is the speed of the liquid at the free surface in absence of waves and $\varepsilon = H/R$ (see Section 3). In cases where the film thickness is much smaller than the cylinder radius as well as the wavelength, Tougou (1977) derived a sequence of asymptotic equations and carried out a linear stability analysis which demonstrated that the cylinder curvature widens the region of instability. Close to the neutral wavenumber a Kurimoto–Sivashinsky equation emerges and a travelling wave solution is identified.

Nonlinear model equations for the film thickness have also been derived to predict the spatio-temporal evolution of the film. These studies have mainly focussed on low volumetric flow rates and are in the low-Reynolds number regime. Frenkel (1992) used lubrication theory to derive a weakly nonlinear equation for the case of a film whose thickness is much less than the cylinder radius; subsequently this was numerically investigated by Kerchman and Frenkel (1994), Kalliadasis and Chang (1994) and Chang and Demekhin (1999). These studies reveal the potentially complex dynamics which the film can exhibit, characterised by the formation of large-amplitude waves that can undergo coalescence. To complement these studies, Kliakhandler et al. (2001) considered the case in which the film thickness is potentially larger than the fibre radius and proposed a model equation for the film thickness that retains the full curvature term. The linear stability characteristics of this equation were shown to be in reasonable agreement with those of the full Navier–Stokes equations; this agreement deteriorated with increasing Reynolds numbers. Their numerical solutions revealed the existence of different flow regimes, which resembled some, but not all, of the regimes observed in experiments conducted as part of their work. In particular, their model could apparently not capture an important regime, often appearing in experiments, namely that which features large drops separated by relatively long flat thin film regions.

More recently, Craster and Matar (2006) have revisited this problem and derived an evolution equation for the total fluid radius based on the assumption that the fluid radius is much smaller than a characteristic, capillary, length scale rather than the fibre radius. This equation is very similar to that used by Kliakhandler et al. (2001). Numerical solutions of the equation derived by Craster and Matar (2006) compared favourably with their experimental observations as well as those of Kliakhandler et al. (2001), in terms of interfacial profiles, droplet spacings and velocities. In particular, regimes involving widely spaced drops were captured successfully.

The modelling studies leading to the evolution equations summarised above have all focused on the case of small volumetric flow rates. For film flows at moderate flow rates, Trifonov (1992) developed an approximate model which is a generalisation of the falling film model derived by Shkadov (1967); the falling film problem and its generalisations still attract considerable attention as the inclined falling film problem benefits from refinements to the basic Shkadov approach (Ruyer-Quil and Manneville, 2002). Trifonov utilised an integral balance (von Karman–Polhausen) approach with an assumed velocity profile that comprised parabolic and logarithmic terms; these reflect gravitational forcing and the significance of cylindrical geometry which becomes important with increasing fluid thickness to cylinder radius ratios. Trifonov (1992) used this model to compute steady travelling waves of two separate 'families'. The model derived by Trifonov (1992), however, exhibits solution non-uniqueness and it is expected that many more families of travelling waves exist.

Motivated by the apparent dearth of studies involving thin film flows on the outside of cylinders at moderate-to-high flow rates, we focus on this case and revisit the Trifonov (1992) model. We solve a re-formulated version of this model in both the linear and nonlinear regimes. In the latter regime, we use bifurcation theory to establish complete understanding of the families of travelling waves that exist for given flow conditions, physical parameters and dimensionless thickness to cylinder radius ratios. These predictions are then compared to transient numerical simulations of the evolution equations, which demonstrate the wave-selection mechanism. We also seek to elucidate the role of inlet forcing on the emerging coherent structures and contrast these with the flow evolution in the absence of forcing.

The rest of this paper is organised as follows: the full problem statement is formulated in Section 2, in which the model equations are derived. A linear stability analysis of this model is carried out in Section 3 and the results are compared to the linear stability characteristics of the full Navier–Stokes equations. Steady travelling waves are presented in Section 4, while the selection of 'dominating' waves from the different wave families computed in Section 4 is demonstrated via transient numerical simulations presented in Section 5. Finally, concluding remarks are provided in Section 6.

2. Mathematical modelling

2.1. Governing equations

We consider the axisymmetric flow of a thin Newtonian, incompressible liquid film, of kinematic viscosity ν, density ρ and constant surface tension σ, down the outside surface of a solid, impermeable cylinder of radius R under the action of gravity g; the cylinder is orientated such that its axis is parallel to the gravitational field. A cylindrical polar coordinate system, $(\tilde{r}, \theta, \tilde{z})$, along with a velocity field $(\tilde{u}_r, 0, \tilde{u}_z)$, are introduced to describe the geometry and dynamics of the fluid film bounded from above by an inviscid gas and from below by a solid cylinder ($\tilde{r} = R$); the free surface is located at $\tilde{r} = R + \tilde{h}$ and so $\tilde{h}$ measures the fluid film thickness.

We will investigate wave regimes in the film using evolution equations derived by Trifonov (1992). These equations, however, will be recast in terms of a generalised falling film model (Shkadov, 1967, 1977). To this end, we use the continuity equa-