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Dynamic modelling of the deformation of a drop in a four-roll mill

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Abstract

The deformation of a drop flowing along the centre streamline of a four-roll mill (4RM) has been investigated. The velocities and elongation rates along the centre streamline in the 4RM were measured using particle tracking velocimetry. The deformation and position of the deforming drops were photographed with a video camera. A dynamic, one-dimensional, analytical simulation model describing the drop deformation has been developed. The model is based on Taylor's [1964. International Congress on Applied Mechanics, vol. 11, 790–796] static conical drop shape model, but has been extended to include elliptic drops undergoing rapid deformation. The model was incorporated into a numerical scheme using Matlab and the drop deformation in the 4RM was simulated. The simulations were compared with the results of the experiments with the help of a dynamic Weber number incorporating the exact effect of the continuous phase stress on the deformation of the drop. With a dynamic Weber number of 0.42 the agreement between the experiments and the simulations along the whole deformation process was excellent for all three drop diameters studied. With this model the deformation of drops of all sizes in different elongation fields can be calculated, for example sub-micron-sized drops in a high-pressure homogeniser.

Keywords: Four-roll mill; Drop deformation; Emulsion; Fluid mechanics; Food processing; Modelling

1. Introduction

In a concentrated emulsion, the shape of the drops, together with their volume fraction and the rheology of the continuous phase, determines the rheological behaviour of the product. One way to create new interesting rheological properties of an emulsion is to alter the shape of the drops (Hamberg, 2003). One of the most basic ways to deform a drop is to stretch it in one dimension. As the forces stretching the drop must arise from the surrounding liquid, it has been found that an elongation flow field in the surrounding liquid results in more efficient stretching of the emulsion drops compared to a pure shear field. (Windhab et al., 2000).

In a process where the emulsion drops are first stretched and then fixed in the stretched shape, the rheological properties of that emulsion can be carefully controlled. In order to gain more insight into the mechanisms controlling drop deformation and to facilitate the construction of an apparatus in which the emulsion drops are stretched and fixed, a model for simulating drop deformation in a varying elongation field has been developed.

Various models have been suggested for the deformation of drops in elongation flow fields, but most of them have been designed for static conditions or very slowly deforming drops (Taylor, 1964; Buckmaster, 1972). The different types of models used include stream function variants, which model the flow around the drop with sources and stresslets (Buckmaster, 1972, 1973; Khakhar and Ottino, 1986), and bounded integral models which solve the integral form of the Stokes' equation at the boundary of the drop (Youngren and Acrivos, 1976). Rallisson (1984) and Stone (1994) both give good reviews of the different models currently in use.

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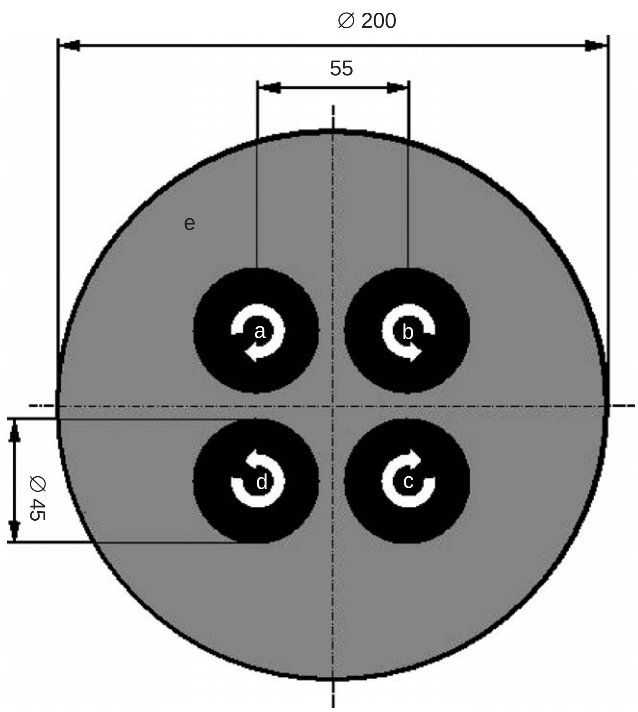

Fig. 1. A schematic illustration of the 4RM. (a)-(d) rolls, (e) sample chamber. Dimensions in mm.

In this study Taylor's (1964) analytical droplet deformation model has been extended to cover dynamic drop deformation. By incorporating the transfer of momentum from the surrounding fluid to the drop into a fitting parameter, called the dynamic Weber number, a very general model has been obtained. As we can prove that the approach is valid, by showing that the dynamic Weber number is independent on the variables in the modelling equation, this model constitutes a powerful simulation tool, which is also very efficient in its use of computer capacity.

The four-roll mill (4RM), Fig. 1, first described by Taylor (1934), is designed to create a fairly constant elongation field in the central region. In this study we used the 4RM to model the up-scalable and process-oriented converging channel, found for example, in a high-pressure homogeniser. To be able to calculate the deformation of the drop it is necessary to know the elongation rate field to which it is exposed. Different authors have performed CFD calculations of the flow and elongation fields in a 4RM (Feng and Leal, 1997; Windhab et al., 2000), but exact measurements seem to be lacking. In this study a particle tracking velocimetry (PTV) method was used to measure the velocity and deformation fields at the centre streamline of the 4RM.

The following approach was used in this study:

- Firstly, a one-dimensional drop deformation model was developed and realised in a Matlab program.
- Secondly, the elongation rate profile was measured in a 4RM.
- Thirdly, the drop deformation was simulated based on the measured elongation rate profile.
- Finally, the simulations were compared with drop deformation measurements, and the dynamic Weber number was defined for the viscosity ratio used.

2. Modelling droplet deformation

2.1. The drop

In this study we focus on the deformation of a spherical drop to a stretched shape in an elongation velocity field varying in magnitude but not in direction, as illustrated in Fig. 2. The length of the drop is denoted as L and the diameter as B. The drop is elongated in the same direction as the elongation rate field (E). The relation between the length and the diameter, the aspect ratio (A), is defined as $A = L/B$. The system studied is a water-in-oil emulsion where the viscosity of the water drops is much lower than the viscosity of the continuous oil phase.

2.2. Models in the literature

Most basic models describing drop deformation and break-up are based on the Weber or the capillary number (Taylor, 1964; Walstra and Smulders, 1998; Janssen and Meijer, 1993). The Weber number describes the relation between the viscous stress on the drop and the Laplace pressure in the drop. Different authors use slightly different numerical constants in the definition.

$$We = \frac{\sigma}{P_L} = \frac{E * \eta_c}{\gamma * (1/r_1 + 1/r_2)}. \tag{1}$$

For a spherical drop $r_1 = r_2 = r$.

$$We = \frac{E * \eta_c}{\gamma * (1/r_1 + 1/r_2)} = \frac{E * \eta_c}{2\gamma/r} = \frac{E * \eta_c * r}{2\gamma}. \tag{2}$$

At a critical Weber number (We_{cr}), depending on many factors, the viscous stress overcomes the Laplace pressure and the drop deforms and breaks.

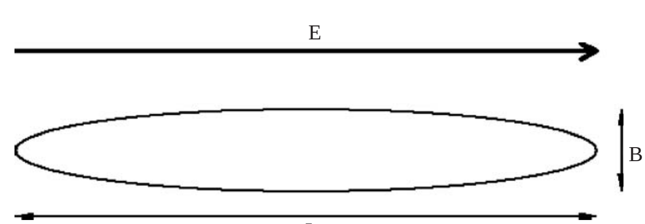

Fig. 2. A schematic illustration of the elliptic/elongated drop. L is the length and B the diameter of the drop. E shows the direction of the elongation field.