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Preparation of cluster states with superconducting qubit network

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Abstract

Based on the architecture of a superconducting charge qubit in [J. Lantz, M. Wallquist, V.S. Shumeiko, G. Wendin, Phys. Rev. B 70 (2004) 140507(R); M. Wallquist, J. Lantz, V.S. Shumeiko, G. Wendin, New J. Phys. 7 (2005) 178], we propose an improved architecture, which can provide a long-range interaction instead of the controllable nearest-neighbor coupling. It can provide two-qubit operation between arbitrary pairs of qubits, which is necessary for the realization of the functional and scalable quantum computing. We further investigate a scheme for generating multi-qubit cluster states which meets the expectations of the so-called one-way quantum computation schemes. It is a simple, scalable and feasible scheme for the generation of cluster states based on the current experiments about the controlled superconducting charge qubit network.

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1. Introduction

Recently, much attention has been paid to the physical realization of a quantum computer, which works on the fundamental quantum mechanical principle. The quantum computer can solve certain hard problems exponentially faster than its classical counterpart. By using unitary quantum logic network, a conventional quantum computer (QC) may be implemented. For realizing quantum computing, some physical systems, such as nuclear magnetic resonance [2], cavity quantum electrodynamics (QED) [3], and optical systems [4], have been proposed. These systems have been demonstrated to possess the advantage of high quantum coherence, but they cannot be integrated easily to form large-scale circuits.

On the other hand, Raussendorf and Briegel recently proposed an intriguing alternative QC strategy, i.e., the one-way quantum computer (QC_C) [5], which constructs quantum logic network by using only single-qubit projective measurements on a generated cluster state [6]. In the QC_C, quantum information is encoded in the cluster state, processed, and read out from the cluster state. The QC_C is universal in the sense that arbitrary unitary quantum logic networks can be carried out based on a suitable generated cluster state. Cluster states thereby serve as a universal source for QC_C. Meanwhile, the cluster states can also be served as entanglement resources [6], which means that other entanglement states can be constructed from the cluster states. As mentioned above, the cluster states have the special characteristics and practical applications, so the preparations of the cluster states have been implemented by many physical systems [7–10].

With the progress of high-precise fabricating technique, superconducting qubits have shown their competence in quantum computing [11,12]. Josephson charge qubit [13–15] and flux qubit [16,17] are based on the macroscopic quantum effects in superconducting circuits. The decoherence time of superconducting qubits is not very long, but the number of quantum operations that can be completed during the coherence time is also comparable with other systems [18]. Owing to its property of large-scale integration [19,20], the superconducting qubits are the promising candidates for scalable quantum computing. In this paper, we propose an alternative and improved scheme for the universal quantum computation and generation of cluster states via the current-controlled superconducting charge qubit network, which can provide two-qubit operation between arbitrary pairs of qubits. It is a simple,

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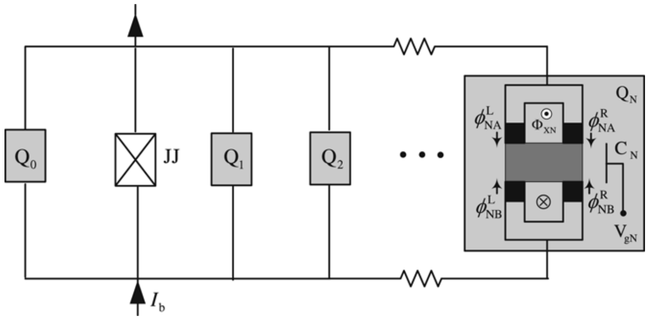

Fig. 1. A current-controlled superconducting charge qubit network structure. All N charge qubits ($Q_1, Q_2, \ldots, Q_N$) can interact with the charge qubit Q_0 by a common large-capacitance Josephson junction (JJ) (denoted as a crossed rectangle). Each charge qubit $Q_k (k = 0, 1, 2, \ldots, N)$ is controlled by a voltage V_{gk} and a local magnetic flux Φ_{Xk}, whereas the coupling of the two qubits is adjusted by the bias current I_b.

scalable and feasible scheme for the generation of the cluster states.

2. Architecture of superconducting qubit network

Since the earliest Josephson charge qubit design [13] was proposed, a series of improved schemes [1,14,18] have been explored. Here, based on the architecture of Josephson charge qubit in Ref. [1], we propose an improved architecture. Ref. [1] only implements the controllable nearest-neighbor coupling, but our improved architecture can provide a physical implementation of two-qubit operation between arbitrary pairs of qubits. The superconducting charge qubit structure is shown in Fig. 1. All N charge qubits ($Q_1, Q_2, \ldots, Q_N$) can interact with the charge qubit Q_0 by a common large-capacitance Josephson junction (JJ) (denoted as a crossed rectangle). For the kth charge qubit, a superconducting island with the induced charge $Q_k = C_k V_{gk} = 2en_k$ is weakly coupled by two symmetric direct current superconducting quantum interference devices (dc SQUIDs) and biased by an applied voltage through a gate capacitance C_k. Assume that all the Josephson junctions of the two symmetric dc SQUIDs have Josephson coupling energy E^0_{Jk} and capacitance C_{Jk}. The self-inductance of each SQUID loop is usually neglected because the loop size (1 μm) is very small. Pierced by a magnetic flux Φ_{Xk}, each SQUID provides an effective coupling energy $-E_{Jk}(\Phi_{Xk})\cos\phi_{kA(B)}$, with $E_{Jk}(\Phi_{Xk}) = 2E^0_{Jk}\cos(\pi\Phi_{Xk}/\Phi_0)$ and the flux quantum $\Phi_0 = h/2e$. The conjugate phase drop $\phi_{kA(B)}$, with subscript $A(B)$ labeling the SQUID above (below) the island, equals the average value $[\phi^L_{kA(B)} + \phi^R_{kA(B)}]/2$ of the phase drops across the two Josephson junctions in the dc SQUID, with superscript $L(R)$ denoting the left (right) Josephson junction.

3. Long-range interaction and universal gates

When $\Phi_{Xk} = \frac{1}{2}\Phi_0$ and $V_{gk} = (2n_k + 1)e/c_k$ for all boxes, except $k = j, 0$, the charge qubit Q_0 and bias current (I_b) coupling JJ only connect to the jth charge qubit, forming a controllable two-qubit system. In the spin-1/2 representation, based on charge states $|0\rangle = |n_k\rangle$ and $|1\rangle = |n_k + 1\rangle$, the effective Hamiltonian of the system becomes [1]

$$H = \sum_{k=0,j} H_k - \frac{\sin^2\left(\frac{\varphi_0}{2}\right)}{E^b_J\cos\varphi_0}\prod_{k=0,j} E^0_{Jk}\cos\left(\pi\frac{\Phi_{Xk}}{\Phi_0}\right)\sigma^{(k)}_x, \quad (1)$$

where $\varphi_0 = \arcsin\frac{\hbar I_b}{2eE^b_J}$, E^b_J is Josephson coupling energy of the coupling JJ, and

$$H_k = \frac{E_{Ck}}{2}(1 - 2n_{gk})\sigma^{(k)}_z - 2E^0_{Jk}\cos\left(\pi\frac{\Phi_{Xk}}{\Phi_0}\right)\cos\frac{\varphi_0}{2}\sigma^{(k)}_x, \quad (2)$$

where $E_{Ck} = (2e)^2/2C_\Sigma$, $C_\Sigma = 4C_{Jk} + C_k$ is the total capacitance of the qubit island and $n_{gk} = C_k V_{gk}/2e$ is the (dimensionless) charge induced on the qubit island by the gate voltage V_{gk}. In the Eq. (1), when bias current $I_b = 0$, any single-qubit operation can be performed by adjusting gate voltage and local magnetic flux Φ_{Xk}. We give the following definitions to simplify our presentation.

$$U^{(k)}_z(\alpha) = \exp[\mathrm{i}\alpha\sigma^{(k)}_z], \quad (3)$$

where $\alpha = -\frac{E_{Ck}}{2}(1 - 2n_{gk})\tau/\hbar$, here, τ is a switching time,

$$U^{(k)}_x(\beta) = \exp[\mathrm{i}\beta\sigma^{(k)}_x], \quad (4)$$

where $\beta = 2\tau E^0_{Jk}\cos(\pi\Phi_{Xk}/\Phi_0)\cos\frac{\varphi_0}{2}/\hbar$.

When both qubits work in their degeneracy points ($V_{gk} = (2n_k+1)e/c_k$), the Hamiltonian of the system in Eq. (1) reduces to

$$H = -2\cos\frac{\varphi_0}{2}\sum_{k=0,j} E^0_{Jk}\cos\left(\pi\frac{\Phi_{Xk}}{\Phi_0}\right)\sigma^{(k)}_x - \frac{\sin^2\left(\frac{\varphi_0}{2}\right)E^0_{J0}E^0_{Jj}\cos\left(\pi\frac{\Phi_{X0}}{\Phi_0}\right)\cos\left(\pi\frac{\Phi_{Xj}}{\Phi_0}\right)}{E^b_J\cos\varphi_0}\sigma^{(0)}_x\sigma^{(j)}_x. \quad (5)$$

For convenience, we give a concise form of Eq. (5) as

$$H = -\overline{E}_{J0}\sigma^{(0)}_x - \overline{E}_{Jj}\sigma^{(j)}_x + C_{0j}\sigma^{(0)}_x\sigma^{(j)}_x, \quad (6)$$

where

$$\overline{E}_{Jk} = 2\cos\frac{\varphi_0}{2}E^0_{Jk}\cos(\pi\Phi_{Xk}/\Phi_0),$$

$$C_{0j} = -\frac{\sin^2(\varphi_0/2)E^0_{J0}E^0_{Jj}\cos(\pi\Phi_{X0}/\Phi_0)\cos(\pi\Phi_{Xj}/\Phi_0)}{E^b_J\cos\varphi_0}.$$

We assume $\overline{E}_{J0} = \overline{E}_{Jj} = C_{0j} = \frac{-\pi\hbar}{4\tau}$ (τ is a given period of time), which can be obtained by suitably adjusting the bias current I_b and magnetic flux Φ_{Xk}. Then Eq. (6) becomes

$$H = \frac{-\pi\hbar}{4\tau}(-\sigma^{(i)}_x - \sigma^{(j)}_x + \sigma^{(i)}_x\sigma^{(j)}_x). \quad (7)$$

From Eq. (7), one can obtain a controlled-phase gate $U_{CPG} = \exp[\mathrm{i}\frac{\pi}{4}(1 - \sigma^{(0)}_x - \sigma^{(j)}_x + \sigma^{(0)}_x\sigma^{(j)}_x)]$, which keeps the two-bit states $|+\rangle_0|+\rangle_j$, $|+\rangle_0|-\rangle_j$, and $|-\rangle_0|+\rangle_j$ unchanged while transforming $|-\rangle_0|-\rangle_j$ to $-|-\rangle_0|-\rangle_j$ when the two qubits are subject to the evolution. Here, $|\pm\rangle$ are defined by $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$. Using $V_j = U^{(j)}_x(-\frac{\pi}{4})U^{(j)}_z(\frac{\pi}{4})U^{(j)}_x(\frac{\pi}{4})$, we implement a controlled-NOT gate $U_{\mathrm{CNOT}} = V^\dagger_j U_{\mathrm{CPG}} V_j$,