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Critical behaviors and magnetic multi-compensation points of bond dilution mixed Blume–Capel model in bimodal magnetic field

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ABSTRACT

Within the effective field theory (EFT), we investigate critical behaviors and magnetic multi-compensation points of bond dilution mixed Blume–Capel model in bimodal magnetic field, for a simple cubic lattice. The tricritical point (TCP) always exists for all h/J values in $T-D$ space, while the TCP appears in smaller or larger range of negative D/J values for $T-h$ space. The bond dilution can depress the TCP. The influence of crystal field and bimodal magnetic field on the induced ordering phase at bond percolation threshold is revealed. The system takes on one or two novel magnetic compensation points in $M-h$ space. The bond dilution has the effect of changing one or two magnetic compensation points. The magnetic multi-compensation points offer a new route in the field of magneto-optic recording.

1. Introduction

The investigation of various Ising models with crystal field has attracted the attention of many works for a long time [1–4]. One of main models is the Blume–Capel model (BCM). In addition to considering the effect of exchange interaction, the BCM also includes longitudinal crystal field in Hamiltonian. A further advantage of this model is that it can be applied to disorder mixed BCM [5–8]. Phase diagrams of disorder mixed BCM provide many fruitful messages, such as reentrant transition, tricritical point (TCP), degeneration of crystal field, induced ordering phase and so on. Some recent research deal with mixed spin model in an external magnetic field. Benayad et al. studied thermodynamic properties of random field mixed spin transverse Ising model [9]. Ekiz studied mixed Ising system in a longitudinal magnetic field [10].

Ferrimagnetic properties of mixed Ising models are another important problem. In a ferrimagnet, experimental studies have shown single or multi-compensation points at which the resultant magnetization vanishes below its Curie temperature [11,12]. In fact, the multi-compensation points went beyond Neel's classification of ferrimagnetic materials. Kaneyoshi reviewed the mutli-compensation temperature points for a decorated square lattice [13]. Very recently, using unique or trimodal magnetic field, BCM found a new magnetic compensation point in $M-h$ space [14,15]. From the theoretical aspects, the mixed Ising models work by using various techniques, such as mean field approximation [16,17], effective field theory [18,19], pair approximation [20], cluster variational method [21], Monte Carlo simulations [22–24] and a precise numerical solution [25–27].

In this work, we study the critical behaviors and magnetic multi-compensation points of bond dilution mixed BCM in bimodal magnetic field. The TCP is always existent for all h/J values in $T-D$ space, while the TCP appears in smaller or larger range of negative D/J values for $T-h$ space. The bond dilution has the effect of depressing the TCP. A smaller crystal field can induce ordering phase at bond percolation threshold. The existence of bimodal magnetic field destroys induced ordering phase. The system displays novel magnetic multi-compensation points in $M-h$ space. Some results were not revealed in previous works.

We note that it is ideal for Monte Carlo simulations to treat pure mixed Ising models. The obtained result is more precise than other procedures. However an obstacle encountered in Monte Carlo simulations on disorder systems is the presence of many metastable states at low temperatures, making it very hard for the

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systems to equilibrate. Hence, the critical behaviors and magnetic properties of disorder mixed spin model at low temperatures are treated by some approximation methods. The expressions for solving present problem are based on the effective field theory (EFT) [28]. Previous research has shown that the results obtained from the EFT are in qualitative agreement with the results of precise numerical solution [27].

The structure of this work is as follows. Section 2 covers the definition of bond dilution mixed BCM in bimodal magnetic field and the theoretical procedures involved. The detailed numerical results and discussions are presented in Section 3, followed by a brief summary.

2. Theory

The Hamiltonian of bond dilution mixed spin-1/2 and spin-1 BCM in a bimodal magnetic field can be defined as:

$$H = -\sum_{\langle i,j\rangle} J_{ij}\sigma_i^z S_j^z - D\sum_j \left(S_j^z\right)^2 + \left(\sum_i h_i\sigma_i^z + \sum_j h_j S_j^z\right). \tag{1}$$

The underlying lattice is composed of two interpenetrating sublattices A and B, where σ_i^z and σ_i^x are spin-1/2 matrices associated with the ith site in sublattice A, while S_j^z and S_j^x are spin-1 matrices associated with the jth site in sublattice B. The first summation is run only over all nearest-neighbor pairs of spins. The second, the third and the fourth summations involve all sites of sublattices A or B. J_{ij} is the exchange interaction between the nearest-neighbor sites, assumed to be $J_{ij} < 0$. D represents an crystal field parameter with the (z) axis. h_α ($\alpha = i$ or j) is a bimodal distribtuion magnetic field to the (z) axis. J_{ij} and h_α can satisfy independent probability distributions, respectively.

$$P(J_{ij}) = p\delta(J_{ij} - J) + (1-p)\,\delta(J_{ij}), \tag{2}$$

$$P(h_\alpha) = \frac{1}{2}\left[\delta(h_\alpha - h) + \delta(h_\alpha + h)\right], \tag{3}$$

where $p_c < p \leq 1.0$. p and p_c denotes bond dilution concentration and bond percolation threshold, respectively. The starting point of the statistics of the system is exactly Callen's identity [29]. Within the EFT, the averaged magnetiations of sublattices A and B are given by

$$\sigma = \langle\langle\sigma_i^z\rangle\rangle_r = \left\langle\left\langle\prod_{j=1}^{z}\left[\left(S_j^z\right)^2\cosh(J_{ij}\nabla) + S_j^z\sinh(J_{ij}\nabla) + 1 - \left(S_j^z\right)^2\right]\right\rangle\right\rangle_r F(x)|_{x=0}, \tag{4}$$

$$m = \left\langle\left\langle S_j^z\right\rangle\right\rangle_r = \left\langle\left\langle\prod_{i=1}^{z}\left[\cosh\left(\frac{1}{2}J_{ij}\nabla\right) + 2\sigma_i^z\sinh\left(\frac{1}{2}J_{ij}\nabla\right)\right]\right\rangle\right\rangle_r G(x)|_{x=0}. \tag{5}$$

The quadrupolar moment q is given by

$$q = \left\langle\left\langle\left(S_j^z\right)^2\right\rangle\right\rangle_r = \left\langle\left\langle\prod_{i=1}^{z}\left[\cosh\left(\frac{1}{2}J_{ij}\nabla\right) + 2\sigma_i^z\sinh\left(\frac{1}{2}J_{ij}\nabla\right)\right]\right\rangle\right\rangle_r L(x)|_{x=0}, \tag{6}$$

where $\nabla = \partial/\partial x$ and z is a lattice coordination number. $\langle\cdots\rangle$ indicates the canonical thermal average, and $\langle\cdots\rangle_r$ denotes the bond dilution average for Eq. (2). The functions $F(x)$, $G(x)$ and $L(x)$ are defined by

$$F(x) = \int P(h_i)\, f(x, h_i)\,\mathrm{d}h_i, \tag{7}$$

$$G(x) = \int P(h_j)\, g(x, h_j)\,\mathrm{d}h_j, \tag{8}$$

$$L(x) = \int P(h_j)\, l(x, h_j)\,\mathrm{d}h_j, \tag{9}$$

while $f(x, h_i)$, $g(x, D_j)$ and $l(x, D_j)$ are as follows:

$$f(x, h_i) = \frac{1}{2}\tanh\left[\frac{\beta}{2}(x + h_i)\right], \tag{10}$$

$$g(x, h_j) = \frac{2\sinh(\beta(x + h_j))}{2\cosh(\beta(x + h_j)) + \mathrm{e}^{-\beta D}}, \tag{11}$$

$$l(x, h_j) = \frac{2\cosh(\beta(x + h_j))}{2\cosh(\beta(x + h_j)) + \mathrm{e}^{-\beta D}}. \tag{12}$$

where $\beta = 1/k_B T$. If we try to treat the spin-spin correlation in Eqs. (4)–(6) exactly, the problem becomes intractable. A cutting approximation is usually adopted:

$$\left\langle\left\langle\sigma_i^z\sigma_j^z\cdots\sigma_l^z\right\rangle\right\rangle \approx \langle\langle\sigma_i^z\rangle\rangle\left\langle\left\langle\sigma_j^z\right\rangle\right\rangle\cdots\langle\langle\sigma_l^z\rangle\rangle, \tag{13}$$

$$\left\langle\left\langle S_j^z\left(S_k^z\right)^2\cdots S_m^z\right\rangle\right\rangle \approx \left\langle\left\langle S_j^z\right\rangle\right\rangle\left\langle\left\langle\left(S_k^z\right)^2\right\rangle\right\rangle\cdots\langle\langle S_m^z\rangle\rangle, \tag{14}$$

for $i \neq j \neq \cdots \neq m$. Then the Eqs. (4)–(6) can be rewritten as

$$\sigma = \left[q\langle\cosh(J_{ij}\nabla)\rangle_r + m\langle\sinh(J_{ij}\nabla)\rangle_r + 1 - q\right]^z F(x)|_{x=0}, \tag{15}$$

$$m = \left[\left\langle\cosh\left(\frac{1}{2}J_{ij}\nabla\right)\right\rangle_r + 2\sigma\left\langle\sinh\left(\frac{1}{2}J_{ij}\nabla\right)\right\rangle_r\right]^z G(x)|_{x=0}, \tag{16}$$

$$q = \left[\left\langle\cosh\left(\frac{1}{2}J_{ij}\nabla\right)\right\rangle_r + 2\sigma\left\langle\sinh\left(\frac{1}{2}J_{ij}\nabla\right)\right\rangle_r\right]^z L(x)|_{x=0}. \tag{17}$$

By combining Eqs. (15)–(17), we get the self-consistent equation with respect to averaged magnetization σ of sublattices A.

$$\sigma = a\sigma + b\sigma^3 + c\sigma^5 + \cdots. \tag{18}$$

Since σ is small enough in the vicinity of the second-order phase transition line, the expressions of second-order phase transition are given by

$$\begin{aligned} a = {} & 2z^2 Q_1[\sinh(J_{ij}\nabla)][R_1[\cosh(J_{ij}\nabla)] + 1 - R_1]^{z-1} \\ & \times F(x)|_{x=1}, = 1, \end{aligned} \tag{19}$$

$$\begin{aligned} b = {} & \frac{4}{3}z^2(z-1)(z-2)Q_2[\sinh(J_{ij}\nabla)][R_1[\cosh(J_{ij}\nabla)] \\ & + 1 - R_1]^{z-1} F(x)|_{x=0} \\ & + \frac{3}{4}z^4(z-1)(z-2)Q_1^3[\sinh(J_{ij}\nabla)]^3[R_1[\cosh(J_{ij}\nabla)] \\ & + 1 - R_1]^{z-3} F(x)|_{x=0} \\ & + 4z^3(z-1)^2 Q_1 R_2[\sinh(J_{ij}\nabla)][\cosh(J_{ij}\nabla) - 1] \\ & \times [R_1[\cosh(J_{ij}\nabla)] + 1 - R_1]^{z-2} F(x)|_{x=0} < 0. \end{aligned} \tag{20}$$

The coefficients Q_1, Q_2, R_1 and R_2 are as follows:

$$Q_1 = \left[\sinh\left(\frac{1}{2}J_{ij}\nabla\right)\right]\left[\cosh\left(\frac{1}{2}J_{ij}\nabla\right)\right]^{z-1} G(x)|_{x=0}, \tag{21}$$

$$Q_2 = \left[\sinh\left(\frac{1}{2}J_{ij}\nabla\right)\right]^3\left[\cosh\left(\frac{1}{2}J_{ij}\nabla\right)\right]^{z-3} G(x)|_{x=0}, \tag{22}$$

$$R_1 = \left[\cosh\left(\frac{1}{2}J_{ij}\nabla\right)\right]^z L(x)|_{x=0}, \tag{23}$$

$$R_2 = \left[\sinh\left(\frac{1}{2}J_{ij}\nabla\right)\right]^2\left[\cosh\left(\frac{1}{2}J_{ij}\nabla\right)\right]^{z-2} L(x)|_{x=0}. \tag{24}$$

On the critical transition line, there may be a line of second-order and a line of first-order transition, separated by a tricritical point (TCP). When $b > 0$, the phase transition is first-order. Hence the point at which $a = 1$ and $b = 0$ determines the TCP on the phase transition line.

On the other hand, the magnetizations σ and m in sublattices A and B are obtained from solutions of the set of coupled equations (15)–(17). The averaged magnetization per site is given by

$$\frac{M}{N} = \frac{1}{2}(\sigma + m), \tag{25}$$