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Gas absorption into a wavy film flowing over a spinning disc

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Abstract

We present results for gas absorption into a liquid film flowing over a spinning disc. The flow is accompanied by the formation of nonlinear waves which strongly influence the diffusion boundary-layer that develops beneath the surface of the film. We use recent advances in modelling of the hydrodynamics and solve a two-dimensional convective-diffusion equation for the solute concentration. Numerical solutions for the finite-amplitude wave regimes and associated integral absorption rates are obtained for flow conditions corresponding to real experiments. Our results show clearly the enhancement of absorption due to the waves.

Keywords: Film flow; Spinning disc; Waves; Gas absorption

1. Introduction

The flow of a thin liquid film over a spinning disc is accompanied by the formation of large amplitude waves which strongly influence the rates of heat and mass transfer processes. This feature is exploited in spinning disc reactors (SDR), which are used in chemical engineering applications such as the manufacturing of pharmaceuticals and fine chemicals. The flow over a spinning disc is related to that involving a film flowing down a vertical (or inclined) plane, the so-called falling film, with notable differences. On the spinning disc, the body force has an azimuthal component and varies radially along the disc. Moreover, the flow of a spinning disc is more controllable than the falling film case, and thus can be exploited more readily for commercial applications.

There are two principal issues related to the experimental and theoretical examination of flow over a spinning disc in the presence of heat and mass transfer effects. The first involves the accurate determination of detailed information regarding surface wave regimes, while the second involves the isolation of the role of waves in increasing, often dramatically, the rates of heat and mass transfer. These issues have already been addressed for the falling film problem (see the recent review by Killion and Garimella, 2001). Here, the waves strongly influence the diffusion of gas into the liquid phase thereby increasing the absorption intensity. A brief review of the results achieved in the investigations relevant to these problems is given below; a more in depth survey is provided in our recent work (Sisoev et al., 2003a, 2005).

The flow over a spinning disc has been the subject of numerous experimental investigations. Of particular importance are the studies conducted by Espig and Hoyle (1965), Charwat et al. (1972), Butuzov and Puhovoi (1976), and Woods (1995). By increasing the flow rate, Q_c, for given liquid properties and rotational speed, Ω, three typical laminar-wave regimes were identified by Espig and Hoyle (1965): a waveless flow with a smooth film thickness, axisymmetric waves travelling towards the disc periphery, and combinations of axisymmetric and helical waves. The experiments of Charwat et al. (1972) demonstrated the formation of axisymmetric or spiral waves, and their decay with increasing radius. Four flow regions along the radius for given flow conditions were identified by Butuzov and Puhovoi (1976): an "inlet" region with a smooth film surface, a "first laminar-wave" region with axisymmetric travelling waves, a "turbulent" region where the film surface is covered by three-dimensional ripples, and a "second laminar-wave" region

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in which these ripples are damped. Careful measurements of the waves were carried out by Woods (1995), who determined the dependence of the wave shapes on the physical parameters. The shapes of typical waves were also studied experimentally by Povarov et al. (1978) and Rifert et al. (1982).

The theoretical investigations of film flow over a spinning disc may be separated into three categories depending on the main subject studied: stationary waveless flow, its linear stability analysis, and nonlinear axisymmetric wave regimes. Investigations of the stationary waveless flow demonstrated the effect of altering the Eckman number, $E = \nu/(\Omega H_c^2)$, on the structure of the solutions; here, H_c is the film thickness scale and ν is the liquid viscosity. Asymptotic solutions for large values of E are available (Rauscher et al., 1973; Shkadov, 1973), and self-similar solutions were subsequently determined (Lepekhin et al., 1981; Shvets et al., 1992). Numerical solutions for finite Eckman numbers were computed within the framework of boundary-layer theory by the finite-difference method (Dorfman, 1967) and by the collocation method (Sisoev et al., 1986). A linear stability analysis of the stationary waveless flow (Charwat et al., 1972; Sisoev and Shkadov, 1987, 1990) revealed the stabilising influence of the Coriolis forces. In the opposite extreme, with increasing E, the parameters of the linear waves approached those characteristic of the falling film problem.

Two approaches were followed in order to derive evolution equations describing the flow of a thin film over a spinning disc in the nonlinear regime. The lubrication approximation (Emslie et al., 1958; Needham and Merkin, 1987) cannot be used to predict the evolution of nonlinear waves measured in experiments since it is valid only for very thin films, when the wave amplitudes are much smaller than their length. Finite-amplitude waves were first modelled for relatively large Eckman numbers in Shkadov (1973) and Rifert et al. (1982). In our recent papers (Sisoev et al., 2003a, 2004; Matar et al., 2004), we have adapted the methods previously developed to study the stability of axisymmetric wave regimes and the nonlinear evolution of finite-amplitude waves on a spinning disc for finite Eckman numbers. The localized version of the evolution equations derived by Sisoev et al. (2003a) allowed us to achieve very good agreement with the experimental results of Woods (1995). The transient computations carried out by Matar et al. (2004) elucidated the mechanism of wave formation and confirmed the results obtained using the localized equations. Apart from establishing the validity of the approach that we have adopted to model the hydrodynamics, the main finding of our work (Sisoev et al., 2003a, 2004; Matar et al., 2004), is that decreasing the value of the Eckman number exerts a stabilizing influence on the finite-amplitude waves, which is in accordance with the results of our linear stability analysis.

The review has so far focused on studies that involved the examination of the hydrodynamics of the flow over a spinning disc. We turn our attention now to investigations that have involved heat and mass transfer effects. The experiments of Aoune and Ramshaw (1999) demonstrated the strong enhancement of the rates of heat and mass transfer into a film flowing over a spinning disc due to the associated formation of waves. The theoretical approach to the problem followed by these investigators, as well as by Rahman and Faghri (1993), utilizes the Nusselt solution, which neglects the presence of the waves. This simple approach does not capture the effects of waves and was shown to significantly under-predict the mass transfer coefficient for wavy flows. The experimental data of Aoune and Ramshaw (1999) are qualitatively consistent with those previously obtained for falling film flow, which is to be expected due to the similarity of these flows. We provide next a brief review of the principal results determined for gas absorption into a falling film.

Experiments involving the absorption of carbon dioxide and oxygen by water films (Emmert and Pigford, 1954; Oliver and Atherinos, 1968) found the absorption rate in wavy films to be approximately 2.5 times greater than that associated with waveless films. Measurements carried out by Jepsen et al. (1966) showed that the appearance of waves on the film surface led to a rapid increase of the gas concentration inside the film. Different approaches to describe the dramatic enhancement of gas absorption into a falling film have been applied by many authors (Ruckenstein and Berbente, 1965; Banerjee et al., 1967; Javdani, 1974; Kholpanov and Shkadov, 1990; Wasden and Dukler, 1990; Bontozoglou, 1998; Roberts and Chang, 2000). The problem was revisited by Sisoev et al. (2005) where the recent advances in film flow modelling were applied to compute the absorption rate through the film surface. The results achieved clearly indicated the strong dependence of the gas absorption rate on the particular wave regime that is realized for a given flow condition. This dependence is explained in terms of the deformation of the diffusion boundary layer by the surface waves.

In this work, we present a theoretical analysis demonstrating the role of the wave regimes on gas absorption into a film flowing over a spinning disc. Since surface tension gradients and associated Marangoni flows are neglected, the hydrodynamic and mass transfer equations decouple. A concise description of the hydrodynamic model employed in this study and the associated solutions is given in Section 2. Details of the formulation of the equations governing the gas absorption process are provided in Section 3. The numerical results are discussed in Section 4. Concluding remarks are provided in Section 5.

2. Film hydrodynamics

2.1. Evolution equations

We consider the flow of a thin, Newtonian, incompressible liquid film, of kinematic viscosity ν, density ρ and surface tension, σ, flowing over a solid, impermeable disc rotating with angular velocity Ω. We use a stationary cylindrical coordinate system, $(\tilde{r}, \theta, \tilde{z})$, in order to describe the film dynamics. The liquid film is bounded from above by an