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Chemical Engineering Science 60 (2005) 4423–4430

Chemical Engineering Science

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Heat transfer phenomena in a solid oxide fuel cell: An analytical approach

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Received 28 May 2003; received in revised form 25 January 2005; accepted 10 March 2005

Abstract

The steady-state heat transfer problem within a fuel cell is considered here. The original 2-D problem is defined while an averaging technique is used in order to reduce its dimensions to one. The full heat transfer equations along the appropriate boundary conditions are analytically solved for both solid and gas phase. Moreover, analytical expressions for the local as well as for the overall heat transfer coefficients are obtained. The gas temperature increases with the distance and the current density, while the temperature of the cell is always of so low variation that can be considered as a constant. Finally, the heat transfer coefficient presents a profile reverse to that of gas temperature.

Keywords: Fuel cell; Heat transfer; Conduction; Convection; Mathematical modeling; Laminar flow

1. Introduction

One of the most promising new technologies in the sector of electrical power systems is that of fuel cells, whose concept has been known for more than a century. Being electrochemical energy conversion devices, the major product of fuel cells is electrical power while their impact on atmosphere quality is zero or near zero. Additionally, the replacement of spontaneous combustion of the typical heat engines with controllable electrochemical oxidation moderates the production of pollutants in the fuel gases. Since the by-products of the electrochemical reaction in a fuel cell are water and a significant amount of heat, effective thermal management is one of the important problems for all the types of fuel cells (Appleby and Foulkes, 1989; Kordesh and Simander, 1996).

Among the different types of fuel cells, solid oxide fuel cells (SOFCs) that are solid state, ceramic cells operating currently at high temperatures, present special interest because of the flexibility in fuel choice due to high operation temperature (Virkar et al., 1997). Since a SOFC stack consists of many connected cells, stack models can be built upon the conventional heat balance equations but further consideration of electrochemical reaction and its consequences should be also taken into account. Usually, the cell stack models require consideration of gas flow, heat transfer, mass transfer, and cell voltage–current relationship. Some mathematical models (e.g. molecular dynamics approach, stochastic models, etc.) need several geometrical and electrochemical parameters, which are functions of temperature and gas composition (Dutta et al., 2001; Koh et al., 2001). Theoretical prediction of temperature distribution is important for SOFC stack design and its stable operation, and it should be achieved by solving a rigorous mathematical model.

A lot of investigations concerning mathematical modeling have been presented in order to adequately simulate the heat transfer within SOFCs. The majority of SOFC thermal models pertain to the planar and monolithic designs (Minh and Takahashi, 1995). Ahmed et al. (1991) assumed fully developed laminar flow presenting Nusselt number value of 3 for convective heat transfer. Bassette et al. (1995) also presented a thermal model of a tubular SOFC design having limited validation to one experimental datum. Kanamura et al. (1989) and Hirano et al. (1992) presented

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0009-2509/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ces.2005.03.013

also thermal models and simulations but of limited electrochemical validation.

This work presents an analytical model for the thermal transport phenomena occurred within a SOFC stack operating at high temperatures. By considering the complete conduction–convection heat transfer equation along the cell, analytical expressions for the spatial distribution of both the gas and the cell temperatures as well as for the local and overall heat transfer coefficient are obtained by using an averaging technique. As the modeled fuel cell is a real cylindrical design and the electrochemistry considered is quite complete, the basic barriers mentioned above are overcome and the use of these analytical formulas can be very helpful in the design and/or in the operating phase of such systems, as the produced results are sufficiently accurate for any practical application.

2. Formulation of the problem

A typical fuel cell stack consists of an anode and a cathode part between which a catalyst (electrolyte) layer exists. The atmospheric air flows in the cathode “area” (gas channel) while the anode is exposed to hydrogen. The present model studies a compatible to the Siemens–Westinghouse cylindrical fuel cell design (George and Bassette, 1998; Hirschenhofer et al., 1998) as it is illustrated in Fig. 1a. By considering the simplest and most studied reaction of hydrogen oxidation in the presence of oxygen (air), one might have the formation of water with heat and electricity production, as follows (Koh et al., 2001):

$$H_2 + \tfrac{1}{2}O_2 \rightarrow H_2O + \text{heat} + \text{electricity}. \tag{1}$$

The area of interest is the cylinder ABCD of Fig. 1a, which is presented in detail in Fig. 1b. By assuming axial symmetry and fully developed incompressible laminar flow and by ignoring heat transfer due to radiation, the conduction–convection heat transport equations can be written as follows:

$$u_z(r)\frac{\partial T_g(r,z)}{\partial z} = \alpha_f\left(\frac{\partial^2 T_g(r,z)}{\partial r^2} + \frac{1}{r}\frac{\partial T_g(r,z)}{\partial r} + \frac{\partial^2 T_g(r,z)}{\partial z^2}\right), \tag{2a}$$

$$s(z) = \alpha_s\left(\frac{\partial^2 T_c(r,z)}{\partial r^2} + \frac{1}{r}\frac{\partial T_c(r,z)}{\partial r} + \frac{\partial^2 T_c(r,z)}{\partial z^2}\right), \tag{2b}$$

where $T_g(r,z)$ and $T_c(r,z)$ are the gas and solid temperature, respectively, $u_z(r)$ is the z-component of the fluid velocity and α_f and α_s are thermal diffusivities in the fluid and the solid phase, respectively.

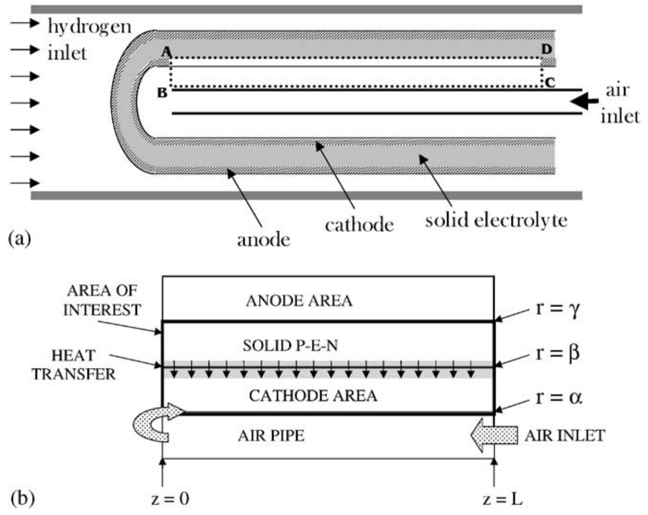

Fig. 1. Design of a tubular fuel cell (a) and blow up of the area of interest (b).

The above equation can be integrated with the following boundary conditions:

$$T_g(r,0) = \text{const.}, \quad \alpha \leqslant r \leqslant \beta, \tag{3a}$$

$$\left.\frac{\partial T_g(r,z)}{\partial z}\right]_{z=L} = 0, \quad \alpha \leqslant r \leqslant \beta, \tag{3b}$$

$$\left.\frac{\partial T_g(r,z)}{\partial r}\right]_{r=\alpha} = 0, \quad 0 \leqslant z \leqslant L, \tag{3c}$$

$$\left.\frac{k(z)}{\rho c_p}\frac{\partial T_g(r,z)}{\partial r}\right]_{r=\beta} = s(z), \quad 0 \leqslant z \leqslant L, \tag{3d}$$

$$T_c(r,0) = \text{const.}, \quad \beta \leqslant r \leqslant \gamma, \tag{3e}$$

$$\left.\frac{\partial T_c(r,z)}{\partial z}\right]_{z=L} = 0, \quad \beta \leqslant r \leqslant \gamma, \tag{3f}$$

$$\left.\frac{\partial T_c(r,z)}{\partial r}\right]_{r=\gamma} = 0, \quad 0 \leqslant z \leqslant L \tag{3g}$$

and

$$T_g(\beta,z) = T_c(\beta,z), \quad 0 \leqslant z \leqslant L. \tag{3h}$$

Boundary conditions (3a) and (3e) insure a constant profile for the gas and solid temperature at the inlet while Eqs. (3b) and (3f) imposes typical von Newman conditions for the outlet temperature. Eqs. (3c) and (3g) are proposed in order to ensure the continuity of the temperature upon the outer boundaries of the area of interest while Eq. (3h) imposes the continuity of the temperature on the gas–solid interface. Finally, Eq. (3d) expresses the transport to the cathode gas of the heat produced upon the electrodic material surface due to electrochemical reaction (1).

By $s(z)$ in Eq. (2b), is denoted the spatial distribution of the heat sources along the cell in the cross-section z, defined as

$$s(z) = j(U_t + j r_{\text{eff}} - E(z)), \tag{4}$$