ELSEVIER

International Journal of Minerals, Metallurgy and Materials
Volume 16, Number 5, October 2009, Page 608

Materials

Band gaps of elastic waves in 1-D phononic crystals with imperfect interfaces

Min Zheng and Pei-jun Wei

School of Applied Science, University of Science and Technology Beijing, Beijing 100083, China
(Received 2008-10-16)

Abstract: Band gaps of elastic waves in 1-D phononic crystals with imperfect interfaces were studied. By using the transfer matrix method (TMM) and the Bloch wave theory in the periodic structure, the dispersion equation was derived for the periodically laminated binary system with imperfect interfaces (the traction vector jumps or the displacement vector jumps). The dispersion equation was solved numerically and wave band gaps were obtained in the Brillouin zone. Band gaps in the case of imperfect interfaces were compared with that in the case of perfect interfaces. The influence of imperfect interfaces on wave band gaps and some interesting phenomena were discussed.

Key words: phononic crystal; transfer matrix; band gap; imperfect interface; Bloch wave

[*This work was financially supported by the National Natural Science Foundation of China (No.10672019).*]

1. Introduction

Phononic crystals, also called acoustic band gap materials, are composite elastic media constituted of periodic repetitions of different solids. The acoustic band gap is one of important properties of phononic crystals. When composite incident waves with various frequencies run into the phononic crystal, the waves with frequency falling into the pass band can propagate through the phononic crystal but the waves with frequency falling into the forbidden band can not propagate through. The filtering property of phononic crystals is why it is called acoustic band gap materials. The potential application of the band gap property in civil and military engineering makes the phononic crystal receive more and more attention in the past ten years [1-7]. It is noted that most of these studies on phononic crystals focused on two aspects. One is the calculation method of band gaps. The other is the forming mechanism of band gaps. In addition, in order to design the desired acoustic band gap, the effect of the topology structure of composites and the contrast of material constants were also investigated extensively [1-2]. The main methods studying phononic crystals at present include the transfer matrix method [6-7], the plane wave expansion method [3], the finite time domain difference method [4], and the multiple scattering method [5]. For one dimensional phononic crystals, the transfer matrix method was usually used due to the laminated feature. The interface in composite materials plays an important role. The interacting of waves and interfaces can influence the wave propagation in a composite material evidently. When the transfer matrix was derived, the continuous condition of displacement vector and traction vector across the interface was usually used. This means the interface is perfect, namely, the joint is fast, which ensures both displacement and traction are continuous across the interface [8-11]. However, the joint between two different solids is often not so fast due to various damages and defects in actual situation [12]. The influence of imperfect interfaces on band gaps is not investigated up to now. In this paper, imperfect interfaces, namely, the displacement or the traction vector jumps across the interfaces, are considered. The influence of imperfect interfaces on wave bands and band gaps is discussed based on the comparison of numerical results obtained from perfect interfaces and imperfect interfaces. The feature of phononic crystals with imperfect interfaces is also investigated.

2. Transfer matrix in the case of perfect

Corresponding author: Pei-jun Wei , **E-mail:** weipj@sas.ustb.edu.cn

Also available online at www.sciencedirect.com

interfaces

Consider a one-dimensional phononic crystal of laminated structure formed by periodic repetition of two different solids, see Fig. 1. The *x* axis is perpendicular to the planar interface of two solids and the *yoz* coordinate plane is parallel to the planar interface. The thicknesses of two solids are a_1 and a_2, respectively. The elastic constants and mass densities of two isotropic solids are denoted by λ_j, μ_j and ρ_j (j=1, 2), respectively.

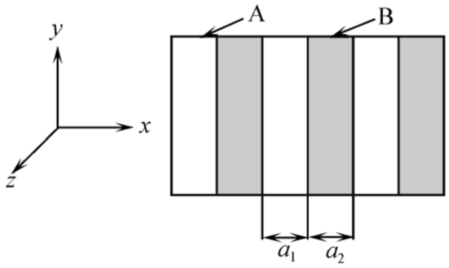

Fig. 1. One dimensional phononic crystal of periodic laminated structure.

For a homogenous, isotropic and linear elastic solid, the motion equation is

$$(\lambda+2\mu)\nabla(\nabla\cdot\boldsymbol{u})-\mu\nabla\times\nabla\times\boldsymbol{u}=\rho\ddot{\boldsymbol{u}} \tag{1}$$

The plane wave propagating along the *x* axis direction, namely, normal incident wave, can be written as

$$\boldsymbol{u}(r,t)=\boldsymbol{u}(r)\mathrm{e}^{\mathrm{i}\omega t}=u(x)\hat{\boldsymbol{e}}\mathrm{e}^{\mathrm{i}\omega t} \tag{2}$$

Inserting Eq. (2) into Eq. (1) leads to

$$\frac{\partial^2 u(x)}{\partial x^2}+k^2u(x)=0 \tag{3}$$

The solution of Eq. (3) is

$$u(x)=A_1\mathrm{e}^{\mathrm{i}kx}+A_2\mathrm{e}^{-\mathrm{i}kx} \tag{4}$$

Therefore, the stress can be obtained by

$$\sigma=\mathrm{i}\rho c^2k\left(A_1\mathrm{e}^{\mathrm{i}kx}-A_2\mathrm{e}^{-\mathrm{i}kx}\right) \tag{5}$$

For incident SH wave,

$$u(x)=u_z(x)\ ,\quad \hat{\boldsymbol{e}}=\hat{\boldsymbol{e}}_z\ ,\quad k=k_{\mathrm{t}}=\omega/c_{\mathrm{t}}\ ,\quad c_{\mathrm{t}}=\sqrt{\mu/\rho}\ ,$$

$$\sigma=\sigma_{xz}=\mu\frac{\partial u}{\partial x}\ .$$

For incident P wave,

$$u(x)=u_x(x),\quad \hat{\boldsymbol{e}}=\hat{\boldsymbol{e}}_x\ ,\quad k=k_1=\omega/c_1\ ,$$

$$c_l=\sqrt{(\lambda+2\mu)/\rho}\ ,\quad \sigma=\sigma_{xx}=(\lambda+2\mu)\frac{\partial u}{\partial x}\ .$$

A perfect interface means the displacement and the traction vectors are continuous along the normal and tangent direction of the planar interface. The boundary condition on the interface can be written as

$$[\boldsymbol{t}]=0\ ,\quad [\boldsymbol{u}]=0 \tag{6}$$

where $[\cdot]$ denotes the jump across interface, $[\boldsymbol{t}]=\boldsymbol{t}^+-\boldsymbol{t}^-$ denotes the traction vector jump, and $[\boldsymbol{u}]=\boldsymbol{u}^+-\boldsymbol{u}^-$ denotes the displacement vector jump. $\boldsymbol{t}^+$, $\boldsymbol{t}^-$, $\boldsymbol{u}^+$ and $\boldsymbol{u}^-$ denote the displacement and traction vectors on both sides of the interface, respectively. The component form of Eq. (6) can be written as

$$\begin{aligned}&\sigma_{xs,1\mathrm{R}}^{(i)}=\sigma_{xs,2\mathrm{L}}^{(i)},\quad u_{s,1\mathrm{R}}^{(i)}=u_{s,2\mathrm{L}}^{(i)}\\&\sigma_{xs,2\mathrm{R}}^{(i)}=\sigma_{xs,1\mathrm{L}}^{(i+1)},\quad u_{s,2\mathrm{R}}^{(i)}=u_{s,1\mathrm{L}}^{(i+1)}\end{aligned} \tag{7}$$

where $s=x,z$. Subscript 1 and 2 denote two different solids (solid A and solid B). Subscript L and R denote the left and right surfaces of each solid. A composite layer constituted of solid A and solid B is called an element layer and is denoted by i (i=1, 2,···, n). Solid A and solid B is called sub-layer (denoted by j(j=1, 2)) of an element layer. The state vector of the left or right surface of each sub-layer is defined as

$$\begin{cases}\boldsymbol{V}_{j\mathrm{L}}^{(i)}=\left\{u_{z,j\mathrm{L}}^{(i)},\sigma_{zx,j\mathrm{L}}^{(i)}\right\}^{\mathrm{T}}\\ \boldsymbol{V}_{j\mathrm{R}}^{(i)}=\left\{u_{z,j\mathrm{R}}^{(i)},\sigma_{zx,j\mathrm{R}}^{(i)}\right\}^{\mathrm{T}}\end{cases}\quad \text{(for SH wave)},$$

$$\begin{cases}\boldsymbol{V}_{j\mathrm{L}}^{(i)}=\left\{u_{x,j\mathrm{L}}^{(i)},\sigma_{xx,j\mathrm{L}}^{(i)}\right\}^{\mathrm{T}}\\ \boldsymbol{V}_{j\mathrm{R}}^{(i)}=\left\{u_{x,j\mathrm{R}}^{(i)},\sigma_{xx,j\mathrm{R}}^{(i)}\right\}^{\mathrm{T}}\end{cases}\quad \text{(for P wave)} \tag{8}$$

State vectors of the left and right surface of the same sub-layer are related by

$$\boldsymbol{V}_{j\mathrm{R}}^{(i)}=\boldsymbol{T}_j'\boldsymbol{V}_{j\mathrm{L}}^{(i)} \tag{9}$$

where $\boldsymbol{T}_j'$ is the transfer matrix of a sub-layer:

$$\boldsymbol{T}_j'=\begin{bmatrix}\cos(k_ja_j) & \sin(k_ja_j)/\omega\rho_jc_j\\ \omega\rho_jc_j\sin(k_ja_j) & \cos(k_ja_j)\end{bmatrix} \tag{10}$$

State vectors of the same surface of adjacent element layers are related by

$$\boldsymbol{V}_j^{(i)}=\boldsymbol{T}_i\boldsymbol{V}_j^{(i-1)} \tag{11}$$

where $\boldsymbol{T}_i$ is the transfer matrix of an element layer. For perfect interface, it can be written as

$$\boldsymbol{T}_i=\boldsymbol{T}_1'\boldsymbol{T}_2' \tag{12}$$

3. Transfer matrix in the case of imperfect interfaces

For the imperfect interface, the displacement and the traction vectors do not satisfy the continuous con-