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Chemical Engineering Science 60 (2005) 2535–2543

Chemical Engineering Science

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Drag force on a circular cylinder midway between two parallel plates at $Re \ll 1$ Part 2: moving uniformly (numerical and experimental)

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Received 13 November 2003; received in revised form 2 August 2004; accepted 2 August 2004

Abstract

To contribute to the determination of the hydrodynamic interactions between a long straight circular cylindrical particle and flow boundaries, we calculate the wall correction of the drag force exerted on a circular cylinder moving uniformly midway between two parallel plane walls, at very low Reynolds numbers. The wall correction factor is numerically and asymptotically investigated. Furthermore, we present a new experimental results for the drag force exerted on this straight circular cylinder. The Navier–Stokes and continuity equations are expressed in the stream function and vorticity formulation and are rewritten in an orthogonal system of curvilinear co-ordinates. These equations are solved with a finite-differences method. The accuracy of the numerical code is tested successfully through a comparison with theoretical and experimental results. In the lubrication regime the numerical calculations of the pressure and viscosity forces are in very good agreement with those obtained by asymptotic expansions. Combining the present results with those obtained in Poiseuille flow (Chem. Eng. Sci. 59 (15, part 1) (2004) 3215) we give the speed at which a force-free cylindrical particle would move with the fluid perpendicularly to it's axis between two planar walls in Poiseuille flow and corrected by wall effects.

Keywords: Hydrodynamics; Laminar flow; Multiphase flow; Suspension; Numerical analysis

1. Introduction

Complex flows of long rod-like particles suspensions (Rahnama et al., 1995; Petrie, 1999; Moses et al., 2001) have important applications in the chemical engineering as processing of composite materials. A theoretical approach of the dynamic of these suspensions presents great difficulties related to the hydrodynamic interactions between these cylindrical particles. A logical first step to understanding many complex phenomena accompanying the sedimentation of rod-like particles is the study of interactions between individual straight circular cylindrical particle or between this particle and flow boundaries at small Reynolds numbers. In fact, these interactions are caused by the long-range velocity distribution generated in the fluid surrounding each moving cylindrical particle, they control their orientation distribution function (Jeffery, 1923). In this paper, we focus on a straight circular cylindrical particle and walls interactions; specifically the effect on the drag force. We assume end effects are ignored (the length of the cylinder is $\gg$ it's radius).

The similar problem concerning the hydrodynamic interactions between solid spherical particles suspension have been the subject of a number of investigations: theoretically, experimentally and more recently numerically (Happel and Brenner, 1973; Batchelor, 1972; Bungay and Brenner, 1973; Ambari et al., 1984, 1985; Feng and Michaelides, 2002).

In the case of suspension of rod-like particles, Cox (1970, 1971) considered the Stokes flow around a circular slender body of length $2L$ and of radius a in the presence of a single solid wall at a distance of order L from this slender body

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doi:10.1016/j.ces.2004.08.050

(in very weak interaction). He calculated the total force and the torque acting on this body as an asymptotic expansion in terms of the body radius to half-length ratio $\alpha = a/L \ll 1$. The same problem was analyzed by Russel et al. (1977) in the situation where a circular slender body is falling near a single vertical wall.

To contribute to the determination of these hydrodynamic interactions, a simplified approach is adopted which consists in considering the case of a circular cylinder of infinite length ($\alpha = 0$) moving uniformly and perpendicularly to it's axis, with constant velocity U_0, without any rotation midway between two parallel plane walls distant of $2b$. In this case, the estimate of these interactions can be done with the calculation of the wall correction factor of the drag force:

$$\lambda(k) = \frac{F_x(k)}{\mu U_0}$$

undergone by the cylinder according to the aspect ratio $k = a/b$. μ is the dynamic viscosity of the fluid.

In this paper we report results concerning the drag force $F_x(k)$ exerted on this cylinder for $0.01 \leqslant k \leqslant 0.99$. It is important to notice that this situation is different from that where we have a single plane. In fact, in the presence of two parallel plates walls we have an additional effect due to the "back-flow" which is confined contrary to the case of a single plane wall where the absence of the second plate facilitates the "back-flow".

For unbounded medium ($k = 0$), Lamb (1911) using a linearization technique of the nonlinear inertia terms suggested by Oseen (1910), gave a first approximate solution of the flow around a cylinder and an analytical expression for the drag force depending on the Reynolds number. Similar calculations have been carried out by many authors as discussed in Part 1 (Ben Richou et al., 2004).

In a semi-infinite viscous liquid bounded by a single plane wall, Takaisi (1955a) discussed the slow motion of a circular cylinder of infinite length perpendicularly to it's axis on the basis of Oseen's linearized equations of motion, assuming that the cylinder is moving parallel to the bounding wall. The formulae for the lift and the drag acting on the cylinder of radius a are obtained to Lamb's approximation in terms of e ($2 \leqslant e = d/a < \infty$), where d is the distance between the cylinder axis and the wall. He showed that the drag force is independent of the Reynolds number (Stokes type solution). In addition, on the basis of the Stokes approximation, Jeffrey and Onishi (1981) studied the slow motion of a cylinder next to a single plane wall in the same configuration. For $1 < e = d/a < \infty$, they gave an exact expression of the drag force. It is obvious that this solution is of the Stokes type.

In confined medium ($k \neq 0$), White (1946) studied the problem by carrying out experiments on cylinder falling perpendicularly to it's axis midway between two vertical plane walls in viscous liquid, and gave an empirical formula for the drag force undergone by this circular cylinder. At sufficiently low Reynolds numbers, there is a very marked difference between White's empirical formula and the well-known Lamb's formula for unbounded fluid, in the sense that the first one is independent of the Reynolds number, while the second depends on it (though both the formulae give the same value for the drag $F_x \to 0$ when $Re \to 0$ or $k \to 0$). For $k \leqslant 0.5$, Faxèn (1946) solved the Stokes's equation for the same problem and gave an asymptotic formula for the drag force. In the same configuration, Takaisi (1955b) solved the Oseen's equation describing the steady slow motion of a circular cylinder at very weak hydrodynamic interactions. He obtained also a drag force independent of Reynolds numbers (Stokes type solution). For $k < 0.2$ the comparison of the results obtained theoretically by Takaisi (1955b) and that obtained experimentally by White (1946) clearly shows a discrepancy. Using finite differences method, Gerald (1997) studied numerically the dynamics of a cylinder sedimenting perpendicularly to it's axis under the influence of gravity in a two-dimensional box filled with viscous fluid at rest. For weak interactions, by extrapolating the numerically obtained terminal velocities to the Stokes' limit for various cylinder diameters, he gave an approximation of the wall correction factor.

Contrary to the case of a cylinder moving uniformly perpendicularly to it's axis in presence of a single plane wall where the bipolar coordinates can be used (Jeffrey and Onishi, 1981), the problem concerning two plane walls is not easy to solve analytically except for weak interactions (Faxèn, 1946; Takaisi, 1955b). To extend the calculations of $\lambda(k)$ in the range $0.5 < k \leqslant 0.99$, we propose in this paper a numerical study, using the stream function ψ and vorticity ω formulation. The grid was carried out by the singularities method (Ben Richou et al., 2004). The successive over-relaxation (SOR) and alternating direction implicit (ADI) techniques are used.

For $0.01 \leqslant k \leqslant 0.99$, we calculated the separate contributions of the dimensionless pressure and viscous drag forces, respectively, λ_p and λ_v ($\lambda(k) = \lambda_p(k) + \lambda_v(k)$). In the lubrication regime we give asymptotic expansions for the correction factors λ , λ_p and λ_v (Ben Richou et al., 2004).

In addition, in this paper we present experimental results on the hydrodynamic drag force exerted on a cylinder and we describe the technique for measurement of this force. The displacement of the cylinder takes place without any rotation midway between the two parallel plane walls. This study was carried out for different low Reynolds numbers and provides a direct verification of our numerical prediction in the range $0.025 \leqslant k \leqslant 0.5$. In this range, our numerical and experimental results are in good agreement with those obtained analytically by Faxèn (1946). In the range $0.9 \leqslant k \leqslant 0.99$, both of the numerical and asymptotic approaches are in good agreement.

Combining the present results with those obtained in Poiseuille flow (Ben Richou et al., 2004) we give the speed at which a force-free cylindrical particle would move with the fluid perpendicularly to it's axis between two planar walls in Poiseuille flow and corrected by wall effects.