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On the modeling of non-Newtonian purely viscous flow through high porosity synthetic foams

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Abstract

In a previous paper a model was proposed for the prediction of non-Newtonian, purely viscous flow, through isotropic high porosity synthetic foams. However, based on work done for creep flow of a Newtonian fluid through two-dimensional arrays of squares, an adaptation to the existing model is discussed and a volumetric partitioning is proposed which facilitates the introduction of possible stagnant regions within the flow domain. Three models, for non-Newtonian purely viscous flow, were derived that allow for various staggering configuration, namely doubly, singly staggered and non-staggered configurations. Results from the proposed model for a doubly staggered configuration compared favorably to experimental pressure gradient data.

Keywords: Non-Newtonian fluids; Porous media; Power law; Pressure drop; Metallic foams

1. Introduction

Smit and Du Plessis (1999) have proposed a model for the prediction of non-Newtonian, purely viscous flow, through isotropic high porosity synthetic foams. The model was derived by volumetric averaging of the equations of motion over a two-phase system of stationary solids and a traversing fluid, yielding (Whitaker, 1967)

$$-\nabla\langle p\rangle = \frac{1}{U_o}\int_{S_{fs}}\int \underline{n}\,p\,\mathrm{d}S - \frac{1}{U_o}\int_{S_{fs}}\int \underline{n}\cdot\underline{\underline{\tau}}\,\mathrm{d}S, \tag{1}$$

where $\langle\,\rangle$ denotes the phase average.

The integral terms in Eq. (1) were evaluated by utilizing a formally introduced *representative unit cell* (RUC), shown in Fig. 1. By assuming that the wall shear stresses τ_w and $\beta\tau_w$ along $S_{||}$ and $S_\perp$, respectively, are uniform and piecewise constant over S_{fs}, the following expression is obtained for the pressure gradient in the Darcy flow regime:

$$-\nabla\langle p\rangle = \frac{S_{||}+\beta S_\perp}{U_o}\,\tau_w\hat{\underline{n}}. \tag{2}$$

The parameter β is the ratio of the magnitudes of the average transverse channel velocity to the average stream-wise channel velocity. For this particular porous medium β can be assumed to be unity.

If $d-d_s$ is the distance between plates and $w=q(\chi/\varepsilon)$ the magnitude of the average velocity in the channel between plates, the wall shear stress for a power-law fluid is given by Smit and Du Plessis (2000) as

$$\tau_w = K\left(\frac{2n+1}{n}\right)^n\left(\frac{\chi}{\varepsilon}\right)^n\left(\frac{2q}{(d-d_s)}\right)^n, \tag{3}$$

where ε is the porosity, K is a consistency index or absolute viscosity (for power-law fluid), and n is a power-law constant. The tortuosity χ, which is the ratio between the average streamline length and the fluid displacement, was given by Smit and Du Plessis (1999) as

$$\frac{\chi}{\varepsilon} \equiv \frac{4}{(3-\chi)^2}. \tag{4}$$

The resulting one-dimensional form of the gradient of the intrinsic phase average pressure for purely viscous power-

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law creep flow may then be expressed as

$$\frac{\mathrm{d}p_f}{\mathrm{d}x} = -\frac{2^{4n+2}3(\chi-1)}{d^{n+1}(3-\chi)^{3n+1}}\left(\frac{2n+1}{n}\right)^n \frac{1}{\chi} K q^n, \tag{5}$$

where d is a macroscopic characteristic length representing the linear dimension of the RUC (Du Plessis et al., 1994) and q is the specific discharge.

The results predicted by Eq. (5) compared fairly well with experimental results (Smit and Du Plessis, 1999), i.e., using the above RUC model (Fig. 1). However, based on work done by Lloyd et al. (2004) for creep flow of a Newtonian fluid through two-dimensional arrays of squares, an adaptation to Eq. (5) is discussed and a volumetric partitioning is proposed which facilitates the introduction of possible stagnant regions within the flow domain.

2. New model

Three RUC models are considered that allow for various staggering configuration, namely doubly staggered shown in Fig. 1 and the singly staggered and non-staggered configurations shown in Fig. 2. The stream-wise volume is denoted by $U_{||}$, the volume for transverse flow by $U_\perp$, the solid volume is given by U_s, and U_g is the stagnant volume.

The two integrals in Eq. (1) are split into stream-wise and transverse integrals, yielding

$$-\nabla\langle p\rangle = \underline{\frac{1}{U_o}\int_{S_{||}}\int \underline{n}\,p\,\mathrm{d}S + \frac{1}{U_o}\int_{S_\perp}\int \underline{n}\,p\,\mathrm{d}S} - \frac{1}{U_o}\int_{S_{||}}\int \underline{n}\cdot\underline{\underline{\tau}}\,\mathrm{d}S - \frac{1}{U_o}\int_{S_\perp}\int \underline{n}\cdot\underline{\underline{\tau}}\,\mathrm{d}S, \tag{6}$$

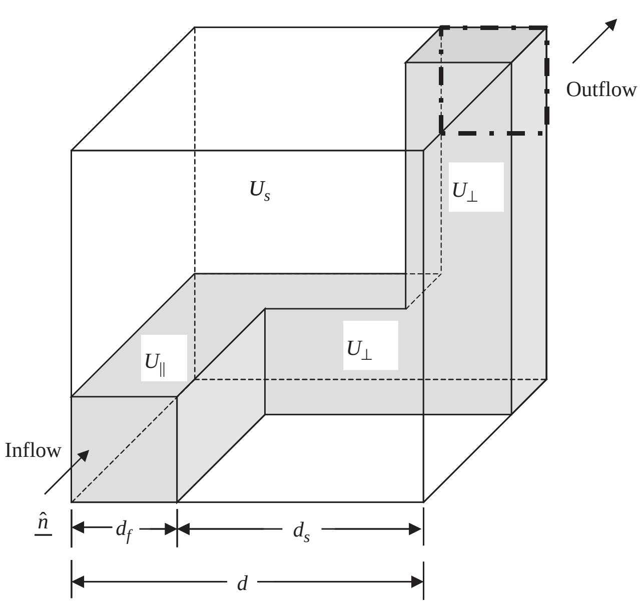

Fig. 1. Representative unit cell, doubly staggered configuration.

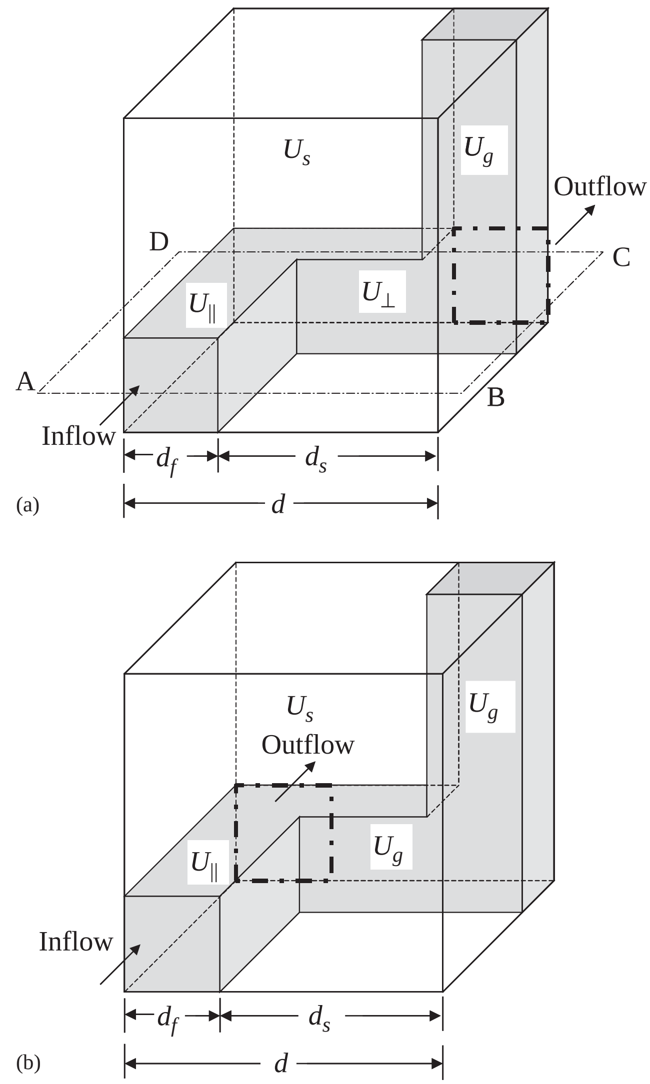

Fig. 2. Singly (a) and non-staggered (b) RUC configurations.

of which the underlined term is zero. The remaining pressure integral in Eq. (6) is split, for the up- and downstream wall faces of $U_\perp$, into a channel wall average pressure, $\bar{p}_w$, and a wall pressure deviation, $\tilde{p}_w$, as follows:

$$\frac{1}{U_0}\int_{S_\perp}\int \underline{n}\,p\,\mathrm{d}S = \frac{1}{U_0}\int_{S_\perp}\int \underline{n}\,\bar{p}_w\,\mathrm{d}S + \frac{1}{U_0}\int_{S_\perp}\int \underline{n}\,\tilde{p}_w\,\mathrm{d}S. \tag{7}$$

An RUC must be a proper substitute for a *representative elementary volume* (REV), of which the boundary cuts through the foam at all possible geometric positions. The surface integrals in Eqs. (6) and (7) must therefore be evaluated over all possible RUCs and since the pressure equation is stream-wise, this will only effect integration over $S_\perp$ planes. For illustration purposes, consider the cross-section between the plane ABCD and the singly staggered RUC shown in Fig. 2. The resulting cross-section, as well as sections of ad-