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# The coefficient of restitution of different representative types of granules

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**Abstract**

Gas fluidised beds have many applications in a wide range of industrial sectors and it is important to be able to predict their performance. This requires, for example, a deeper appreciation of the flow of the particles in such systems using both empirical and numerical methods. The coefficient of restitution is an important collisional parameter that is used in some granular flow models in order to predict the velocities and positions of the particles in fluidised beds. The current paper reports experimental data involving the coefficients of restitution of three different representative types of granule viz. melt, wet and binderless granules. They were measured at various impact velocities and the values were compared with those calculated from different theoretical models based on quasi-static contact mechanics. This required knowledge of the Young's moduli and yield stresses, which were measured quasi-statically using diametric compression. The results show that the current theoretical models for the coefficient of restitution explored here lead to either an over- or an under-estimation of the measured values. The melt granules exhibited the greatest values of the coefficient of restitution, Young's modulus and yield stress. The differences in these values were consistent with the nature of the interparticle bonding for each of the three granule types. A new model for the calculation of the coefficient of restitution of granular material was developed that takes account of the work hardening of the granules during impact. Generally, this model provides an improved prediction of the measured values.


*Keywords:* Young's modulus; Yield stress; Coefficient of restitution; Critical yield velocity; Binderless granule; Wet; Melt; Autoadhesion

## 1. Introduction

Gas fluidised beds have many applications in different sectors of the chemical processing industries (Kunii and Levenspiel, 1991) because of their numerous advantageous characteristics including isothermal conditions throughout the bed, excellent heat and mass transfer, and the possibility of continuous operation. Traditional applications include those employed in the petrochemical, pharmaceutical, chemical, metallurgical and energy industries. In the chemical and the pharmaceutical industries, 'fluidised bed granulation' is an important application that transforms powdery material into granules or agglomerates. Granules have superior properties when compared to the feed powder, for instance improved handling and flowability (Hapgood, 2000). The process is based on introducing an atomised liquid binder by means of a binary nozzle with pressurised air. Particle growth occurs because liquid droplets are deposited onto the bed of fluidised particles and they are bound together initially by a combination of surface tension, capillary pressure and viscous forces (Iveson and Litster, 1998). Generally, the liquid is a solution of a solid binder or it is a melt so that the granules are stabilised by evaporation or cooling. The granules move vigorously in a fluidised bed, which results in collisions with other granules or the walls of the equipment. The disruptive forces due to this mixing action are critical for achieving the correct balance with the binding forces, so that the size of the granules may be controlled. There is an extensive literature on predicting the performance of industrial fluidised beds involving numerical simulation as will be discussed later. This involves computing the flow of the particles but the collisions occurring in the system and the random nature of the motion of the particles mean that it is a significant challenge to make realistic predictions.

The distinct element method (DEM), sometimes termed granular dynamics (GD), is the most commonly used numerical procedure for simulating the behaviour of particulate

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systems. There are two main schemes involving either hard or soft body interactions. The hard body scheme is based on instantaneous collisions that are specified by both normal and tangential coefficients of restitution together with a coefficient of friction (Hoomans, 2000); the coefficient of restitution is the ratio of the rebound and impact velocities. This scheme is computationally more efficient than those based on soft body interactions but there are some limitations, for instance, only two-body collisions may be treated in a given time step. A more detailed understanding at the micro-scale is possible by the application of one of the soft body methods because the contact forces are physically sensible (see for example, Couroyer et al., 2000; Golchert et al., 2004; Mishra et al., 2002; Grantt et al., 2005; Thornton and Antony, 2000; Thornton and Liu, 2004, Kafui and Thornton, 2000; Thornton et al., 2004).

As demonstrated by Goldschmidt et al. (2001), the coefficient of restitution of particles was shown to influence the hydrodynamics of dense gas-fluidised beds using a two-dimensional multi-fluid Eulerian computational fluid dynamics (CFD) model. It was observed that a decrease in the coefficient results in a smaller number of elastic collisions generating greater fluctuations in the kinetic energy. Later, a comparison was made between the experimental results obtained with glass beads at different fluidising velocities and numerical simulation data, which were generated assuming a hard sphere discrete particle model and a two-fluid continuum model with closures according to the kinetic theory of granular flow (Goldschmidt et al., 2004). Close agreement was achieved when the coefficient of restitution was re-defined as an effective parameter that accounted for additional dissipation due to frictional interactions (Goldschmidt et al., 2004). Moreover, Taghipour et al. (2005) used the coefficient of restitution to characterise the solid-phase kinetic energy fluctuations in a gas-solid fluidised bed granulator. Zhang et al. (2005) employed the coefficient of restitution as a parameter for evaluating the fluid lubrication effect during approach and separation of granules by incorporating a Stokes number.

The coefficient of restitution, which is a function of the geometry of the colliding bodies and their relative velocities, is a measure of the fraction of kinetic energy recovered during a collision. The energy can be dissipated as a result of stress wave propagation, plastic deformation or viscoelastic phenomena (Siefried et al., 2005). If an impact is perfectly elastic, energy is dissipated by the propagation of elastic waves. Hunter (1957) showed theoretically that this was less than 1% of the initial kinetic energy for the impact of a steel ball on a large block of steel or glass. Dahneke (1971) calculated that the coefficient of restitution of elastic collisions was in the range 98–99%, implying that the energy loss by elastic waves is less than 1% or 2% of the total impact energy.

Plastic deformation is an important dissipation mechanism at relatively high velocities for some materials. Several theoretical analyses have shown that the coefficient of restitution can be predicted using contact mechanics based on assuming that the impact loading is perfectly plastic and treating the rebound as perfectly elastic (Johnson, 1985; Thornton, 1997). In general, it was shown that the coefficient of restitution decreases with increasing velocity. However, at large deformations it decreases at a faster rate (Wu et al., 2003) and, if the particles are adhesive, a maximum occurs at some intermediate impact velocity (Thornton and Ning, 1998; Fu et al., 2004a,b). Gorham and Kharaz (2000) found that the theories of Johnson (1985) and Thornton (1997) were consistent with the rebound of precision spheres impacting a flat platen at impact velocities that were at least a factor of 50–100 greater than the velocity corresponding to the onset of plastic yield in the platen. More recently it has been shown that it may be important to account for the initial elastic strain during the loading stage of the impact (Wu et al., 2005).

While there has been numerous theoretical and numerical studies on the impact behaviour of homogeneous spherical particles including for example, Vu-Qouc and Zhang (1999), Zhang and Vu-Quoc (2002) and Adams et al. (2004), there has been relatively little work on the calculation of the coefficient of restitution of granules, which have heterogeneous internal structures of a discrete nature. It is also the case that only relatively few measured values of the coefficient of restitution have been reported, for example, wet granules (Fu et al., 2003) and binderless polystyrene granules (Cheong et al., 2005). In the absence of either measured or calculated values of the coefficient of restitution, most numerical studies of fluidised beds have assumed a constant value (Goldschmidt et al., 2001; Goldschmidt et al., 2004; Taghipour et al., 2005; Zhang, 2005), which involves significant uncertainties given the velocity dependency of this parameter.

The present work attempts to address the limited availability of experimental data by investigating the impact behaviour of three representative types of granules viz. wet, melt and binderless granules that are encountered in fluidised bed operations. In this work *wet granules* are defined as granules in which the primary particles are held together by liquid bridges; the *melt granules* are wet granules in with solidified binder. The *binderless granules* are granules without binder. Obviously, the behaviour of *wet granules* is important in determining the granular flow pattern in a fluidised bed granulator or drier when the binder still exists as fluid. The flow pattern is then dictated by the impact response of *melt granules* towards the end of a fluidised bed drying process as the liquid binder solidifies to form solid bonds. Apart from this, the potential of manufacturing binderless granules for inhalation formulation using a pressure swing fluidised bed has been identified by Horio (2003). To provide more insight into this process, it is interesting to study the impact behaviour of *binderless granules* comprising constituent particles held together by short-range forces such as van der Waals interactions.

In some detergent industries, it has become common practice to combine high shear granulation with lower shear granulation (fluidised bed granulation) techniques in the manufacturing process in order to produce high density granules (Rough et al., 2003; Rough et al., 2005; Mort, 2005; Mort et al., 2001). Fine granules are produced using high shear granulation followed by low shear continuous granulation. For this reason, all the granules examined in the current work were granulated in a high shear mixer. They were composed of Durcal