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Evaluating the performance of a cascade of two bioreactors

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Abstract

We investigate a reactor network consisting of two chemostats in series. Previous researchers have compared the performance of a two-reactor system against a single reactor having the same total residence time. In this paper, we suggest that it is more natural to compare the performance of a cascade against the optimal performance a single-reactor system having the same, or smaller, residence time.

We consider a biological system in which the growth rate is given by a Monod expression with a variable yield coefficient. We find that it is possible for this model to obtain a significant increase in performance by using a two-reactor system. However for the two-reactor system the performance enhancements are achieved when the system reaches a time-invariant steady-state rather than under conditions which produce self-generated oscillations, which was the focus of interest of earlier researchers.

Keywords: Bifurcation; Bioreactors; Modelling; Nonlinear dynamics; Reaction engineering; Stability

1. Introduction

The past four decades have seen extensive research aimed at improving product yields in chemical reactors. Many studies, both experimental and theoretical, have shown that periodic forcing is an appropriate engineering tool to improve the conversion or selectivity of a desired product (Silveston et al., 1995; Stankiewicz and Kuczynski, 1995). However, the additional complications and costs associated with implementing external periodic operation have limited the industrial uptake of this technique (Silveston et al., 1995; Stankiewicz and Kuczynski, 1995).

The possibility of combining the advantages of periodic operation with the benefits of using two reactors arranged in series through the use of 'natural oscillations' has been investigated by several authors (Balakrishnan and Yang, 1998; Chen et al., 1995; Jianqiang and Ray, 2000; Ray, 1995; Yang and Su, 1993). By 'natural oscillations' it is meant that the process parameters are chosen so that a *steady input* of reactants into the first reactor generates *self-sustained oscillations* in its output. This output then forces the second reactor. The attraction of this method is that no external energy is required to generate the oscillations. Improvements in reactor performance are therefore achieved without the additional costs associated with external periodic forcing. Consequently, this approach harnesses the advantages of periodic forcing without the expense of its implementation. Significant increases in product yields for various biochemical processes using this approach have been shown to be theoretically possible (Balakrishnan and Yang, 1998; Chen et al., 1995; Jianqiang and Ray, 2000; Yang and Su, 1993).

1.1. Review of related work

In this paper, we re-investigate a model system of two continuous-flow tank bioreactors (CSTB) in which the specific growth rate is given by a Monod expression with a variable yield coefficient. This system was previously considered by Balakrishnan and Yang (1998), Chen et al. (1995) and Yang and Su (1993). These authors investigated how the performance of a two-reactor system changes as the residence time in the first reactor is varied, assuming that the total residence time of the system is fixed.

Yang and Su (1993) considered the situation when the residence times in each reactor were equal as their baseline. For

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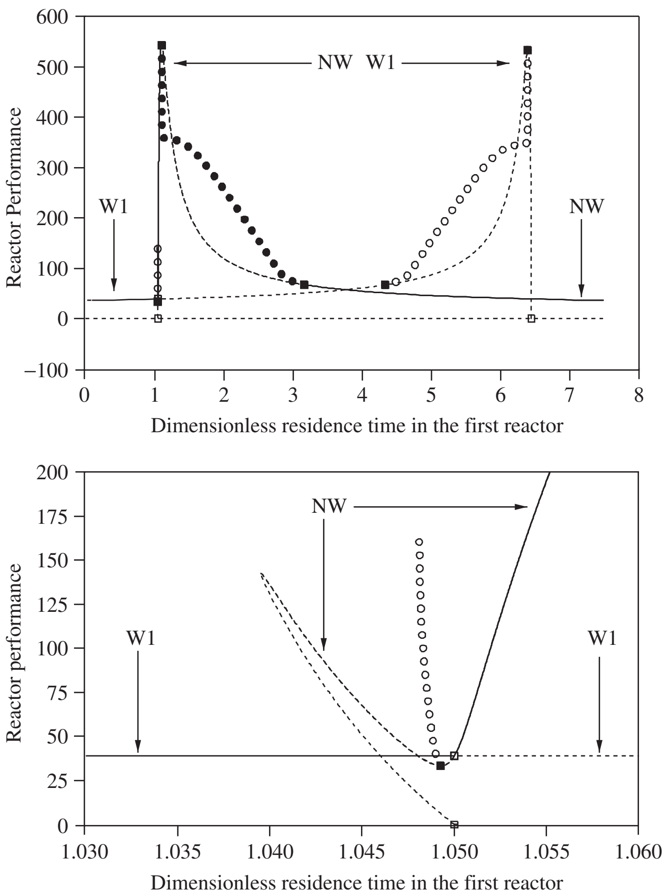

Fig. 1. Steady-state diagram showing the variation of reactor performance (X_2^*) with dimensionless residence time in the first reactor (τ_1^*). Solid lines are stable steady states, dotted lines are unstable steady states; open squares are branch points; filled in squares are Hopf bifurcation points; open circles are unstable periodic orbits; and, filled-in circles are stable periodic solutions. The stable part of these figures were obtained by Balakrishnan and Yang (1998) and Yang and Su (1993) by integrating the governing equations. Nomenclature: W1 denotes a branch of solutions in which washout occurs in the first reactor; NW denotes a branch of solutions in which washout does not occur in the first reactor. Parameter values: $S_0^* = 20$, $\tau_t^* = 7.5$.

the parameter values used in our Fig. 1 they noted that there was "more than six-fold (6.17) increase in performance" as the residence time in the first reactor was varied. For the same parameter values they also noted that, regardless of the choice of residence time in the first reactor, "the performance of a two-CSTB-in-series system is always better than that of one CSTB of equal dilution rate". They concluded that the operation of "two CSTBs connected in series, can lead to enormous improvement in their performances".

Motivated by the work of Yang and Su (1993), Chen et al. (1995) investigated the performance enhancement in a two CSTB system due to the generation of natural oscillations in the first CSTB. For the case of a single CSTB they found that "the values of time-averaged performance of the system operated in the oscillatory region is always less than that operated in the optimal steady state" for the Monod, Tessier and Moser growth models. As the optimal steady-state was always stable these authors concluded that "it is not profitable to split the bioreactor into two or more smaller ones" for these types of models. Subsequently, they analysed a biological system whose growth kinetics involves substrate inhibition and the performance of the two-reactor system was maximised and compared against a single reactor having the same total residence time.

Balakrishnan and Yang (1998) revisited the Monod-growth model whilst investigating two more complex microbial systems. The authors "speculate that the performance of a two-chemostat-in-series system may be in general be better than a single chemostat system with the same total residence time". This comment contradicts the conclusion stated earlier by Chen et al. (1995). For the more complex systems, Balakrishnan and Yang (1998) stated that "the two-chemostat-in-series system performed better than a single chemostat of the same total residence time".

In this paper, we re-investigate the Monod growth model with a variable yield coefficient in both the single and the double CSTB cases. We study how the performance of these systems depend on the substrate concentration and residence times. A critical issue in assessing the performance of the cascade configuration is the determination of a suitable criterion for comparing the performances between such a system with that of a single CSTB. We believe that comparisons between the two configurations should not be done using the same operating conditions (such as total residence times), but rather with the best possible performances for each reactor configuration. With such a definition, we are able to determine the values of the feed substrate concentration and design arrangements for the cascade which result in the most efficient performance when compared to a single CSTB.

2. Model equations

We investigate a microbial system in which cell mass (X_i, $i = 1, 2$) grows through consumption of a substrate species (S_i). The specific growth rate, Eq. (5), is given by a Monod expression with variable yield coefficient, Eq. (6), but without product and substrate inhibition. The problem is to maximise the cell mass concentration leaving the reactor (X_2) as a function of the residence time in the first reactor (τ_1) for a given total residence time in the two reactors (τ_t). This microbial system in two reactors has been investigated by Balakrishnan and Yang (1998) and Yang and Su (1993). In a single reactor it has investigated by Balakrishnan and Yang (2002) and Chen et al. (1995).

The dimensional and dimensionless forms of our model are stated in Sections 2.1 and 2.2, respectively.

2.1. Dimensional model

Following Yang and Su (1993), the governing equations of our system are given by

Reactor 1:

$$V_1 \frac{dS_1}{dt} = F(S_0 - S_1) - V_1 X_1 \frac{\mu(S_1)}{Y(S_1)}, \tag{1}$$

$$V_1 \frac{dX_1}{dt} = F(X_0 - X_1) + V_1 X_1 \mu(S_1), \tag{2}$$