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Chemical Engineering Science 61 (2006) 1932–1945

Chemical Engineering Science

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Low-Reynolds-number motion of a heavy sphere between two parallel plane walls

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Received 18 July 2005; received in revised form 29 September 2005; accepted 19 October 2005
Available online 9 December 2005

Abstract

A novel boundary-integral algorithm [Staben, M.E., Zinchenko, A.Z., Davis, R.H., 2003. Motion of a particle between two parallel plane walls in low-Reynolds-number Poiseuille flow. Physics of Fluids 15, 1711–1733; Erratum: Phys. Fluids 16, 4206] is used to obtain O(1)-nonsingular terms that are combined with two-wall lubrication asymptotic terms to give resistance coefficients for near-contact or contact motion of a heavy sphere translating and rotating between two parallel plane walls in a Poiseuille flow. These resistance coefficients are used to describe the sphere's motion for two cases: a heavy sphere driven by a Poiseuille flow in a horizontal channel and a heavy sphere settling due to gravity through a quiescent fluid in an inclined channel. When the heavy sphere contacts a wall in either system, which occurs when the gap between the sphere and the wall becomes equal to the surface roughness of the sphere (or plane), a contact-force model using the two-wall resistance coefficients is employed. For a heavy sphere in a Poiseuille flow, experiments were performed using polystyrene particles with diameters 10%–60% of the channel depth, driven through a glass microchannel using a syringe pump. The measured translational velocities for these particles show good agreement with theoretical results. The predicted translational velocity increases for increasing particle diameter, as the spheres extend further into the Poiseuille flow, except for particles that are so large (diameters of 80%–85% of the channel depth) that the upper wall has a dominant influence on the particle velocity. For a heavy sphere settling in a quiescent fluid in an inclined channel, the transition from the no-slip regime to slipping motion occurs for a larger inclination angle of the channel with respect to the horizontal for an increase in particle diameter, since the larger particles are more slowed by the second wall. Limited experiments were performed for Teflon spheres with diameters 64%–95% of the channel depth settling in a very viscous fluid along the lower wall of an inclined acrylic channel. The measured translational velocities, which are only about 15%–25% of the tangential component of the undisturbed Stokes settling velocity, are in close agreement with theory using physical parameters obtained from similar experiments with a single wall [Galvin, K.P., Zhao, Y., Davis, R.H., 2001. Time-averaged hydrodynamic roughness of a noncolloidal sphere in low Reynolds number motion down an inclined plane. Physics of Fluids 13, 3108–3119].

Keywords: Colloidal phenomena; Fluid mechanics; Hydrodynamics; Laminar flow

1. Introduction

Particle transport at low Reynolds number in confined geometries is relevant to many applications, including sedimentation (Ambari et al., 1993), blood flow (Brenner and Bungay, 1971; El-Kareh and Secomb, 2000), and suspensions processing (Davis, 1993). The recent development of microfluidics, generally defined as the movement of fluid in channels with cross-sectional dimensions on the order of tens to hundreds of microns and used for various biological and chemical applications, has only increased interest in low-Reynolds-number transport research. Particulate transport in microfluidic devices is important for various microfluidics applications, such as flow cytometry (Eyal and Quake, 2002; Jackson et al., 2002; Yao et al., 2004). In many cases, the problem of interest is the motion of a spherical particle between two plane walls. Heavy (or light, compared to the surrounding fluid) spheres may contact one of the walls of the channel in a finite amount of time, assuming that the walls and/or sphere are not perfectly smooth, in which case contact forces need to be included and the sphere may be considerably slowed by interactions with the wall.

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0009-2509/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ces.2005.10.041

Many researchers have investigated the problem of a sphere between two parallel plane walls, with much of the work prior to 1965 reviewed by Happel and Brenner (1986). More recently, Ganatos et al. (1980a,b) used a collocation method to study the force and torque on a sphere in creeping flow between two plane walls for motion both perpendicular and parallel to the walls, and then combined these results from the collocation method (Ganatos et al., 1980a,b) with the one-wall lubrication asymptotic formulations of Goldman et al. (1967a,b) to handle smaller separations between the particle and the wall (Ganatos et al., 1982). However, the accuracy of the latter work of Ganatos et al. (1982) was limited by the minimum gaps of 10% of the particle radius obtained in the earlier work (Ganatos et al., 1980a,b). Staben et al. (2003) developed a boundary-integral formulation yielding accurate results for the translational and rotational velocities for very small sphere-wall separations of less than 1% of the particle radius. These boundary-integral results were combined with a two-wall resistance formulation, using the near-field asymptotic formulas of Goldman et al. (1967a,b), to describe the motion of neutrally-buoyant spheres for arbitrarily small particle-wall separations.

Often, the particle being studied is not neutrally buoyant, e.g., a heavy particle settling along an inclined channel (Galvin et al., 2001; Romero et al., 1993). In these situations, the particle will closely approach a wall, and microscopic surface roughness on the particle and/or wall may lead to solid–solid contact. Asymptotic lubrication solutions for a sphere moving parallel to a single wall, which are applicable for near-contact motion, were developed by Goldman et al. (1967a,b), while the corresponding lubrication solutions for motion of a sphere perpendicular to a plane wall were obtained by Cooley and O'Neill (1969). Staben et al. (2003) used the near-contact formulas of Goldman et al. (1967a,b) to develop lubrication asymptotic formulations for a neutrally-buoyant sphere in a Poiseuille flow between two plane walls.

In the works discussed above, the interaction once the sphere contacts a wall was not addressed. Lubrication theory indicates that two perfectly smooth surfaces in close approach in a continuum liquid would never physically come in contact under the action of a finite force (Batchelor, 2002; Reynolds, 1886). van der Waals and other nonhydrodynamic forces that increase with decreasing surface–surface separation can lead to contact of perfectly smooth surfaces. However, for the particle diameters employed in the present work (5 μm and larger), hydrodynamic forces are generally at least one order-of-magnitude greater than the nonhydrodynamic forces between the sphere and plane (c.f., Suresh and Walz, 1996). Then, a smooth sphere sedimenting towards a smooth, inclined plane surface will asymptotically approach the plane and move downwards along it due to gravity, without physical contact. However, experiments by Carty (1957) showed a small but constant separation between the sphere and the plane that is not predicted by lubrication theory, for which Goldman et al. (1967a,b) proposed surface roughness as a possible explanation. For microscopically rough surfaces, solid–solid contact forces can occur between the sphere and plane, once the nominal separation decreases to the maximum roughness height, breaking the symmetry of the lubrication formulas given by Goldman et al. (1967a,b). Based on these findings, Smart et al. (1993) developed a contact-force model for sphere motion along an inclined plane, incorporating a single surface roughness height on the sphere to describe solid–solid contacts between the rough sphere and the smooth plane. Good agreement was observed between theory and experiments for particles with a uniform roughness height.

Prokunin (1996) performed experiments at low Reynolds number for a sphere sliding down the inner surface of a tube. For large inclinations of the tube, the sphere attained a constant distance from the tube wall that was considerably larger than the average surface roughness height, perhaps due to the presence of multiple roughness heights. Galvin et al. (2001) and Zhao et al. (2002) developed a theoretical model with two roughness heights and obtained a time-averaged measure of the hydrodynamic roughness of the sphere in contact with an inclined plane. The small asperities are sufficiently profuse as to cause the particle to maintain at least a minimal separation from the plane, while the larger asperities are more sparsely distributed. The large asperities were found to have a much more significant effect on the time-averaged hydrodynamic roughness at large angles of inclination of the plane, increasing the average separation of the sphere from the plane.

The motion of a spherical particle in contact with a plane is of interest in the rapidly growing field of microfluidics. Fluid flow in microfluidic devices is often driven by pumps, either found on the device (Good et al., 2004) or as an external attachment to the device (Jackson et al., 2002). Spherical particle transport in microchannels is quite relevant to many cell-based applications (Chen et al., 2004), as well as in particle-based assays (Verpoorte, 2003). Since cells and particles are generally denser than the buffer or other fluid in which they are transported, they are likely to settle and contact the lower wall of a microchannel in a finite time, in which case a contact-force model is needed to describe the motion.

Our goal in this work is to develop a simple model for analyzing non-neutrally-buoyant spherical particle motion in a parallel-plate channel at low Reynolds number. We use a previously-developed boundary-integral algorithm (Staben et al., 2003) to obtain the O(1)-nonsingular terms of the resistance coefficients, which are functions only of the ratio of the sphere diameter and channel height, for motion perpendicular and parallel to the walls. Resistance coefficients are then obtained using these functions and the known logarithmic terms for near-contact hydrodynamic interactions of a sphere and a plane wall. The basic resistance formulation for the motion of a non-neutrally-buoyant spherical particle between two walls is separated into two cases: a heavy sphere driven by a Poiseuille flow in a horizontal channel and a heavy sphere settling in an inclined channel in a quiescent fluid. A contact-force model with a single asperity height on the sphere (or plane) is used to describe the motion of the sphere when it contacts the wall, i.e., when the gap between the sphere and wall is equal to the surface roughness height. Experimental results are obtained for the motion of a heavy sphere driven by a syringe pump along the lower wall of a horizontal microfluidic channel and for a