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Chemical Engineering Science 60 (2005) 5480–5498

Chemical Engineering Science

www.elsevier.com/locate/ces

A generalized analysis on material invariant characteristics for microwave heating of slabs

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Received 13 January 2005; received in revised form 19 April 2005; accepted 25 April 2005
Available online 28 June 2005

Abstract

A generalized dimensionless formulation has been developed to predict the spatial distribution of microwave power and temperature. The 'dimensionless analysis' is mainly based on three numbers: wave number, N_w; free space wave number, N_{w0}; and penetration number, N_p, where N_w is the ratio of sample thickness to wavelength of microwaves within a material, N_{w0} is based on wavelength within free space and N_p is the ratio of sample thickness to penetration depth. The material dielectric properties and sample thicknesses form the basis of these dimensionless numbers. The volumetric heat source due to microwaves can be expressed as a combination of dimensionless numbers and electric field distributions. The spatial distributions of microwave power for uniform plane waves can be obtained from the combination of transmitted and reflected waves within a material. Microwave heating characteristics are obtained by solving energy balance equations where the dimensionless temperature is scaled with respect to incident microwave intensity. The generalized trends of microwave power absorption are illustrated via average power plots as a function of N_w, N_p and N_{w0}. The average power contours exhibit oscillatory behavior with N_w corresponding to smaller N_p for smaller values of N_{w0}. The spatial distributions of dimensionless electric fields and power are obtained for various N_w and N_p. The spatial resonance or maxima on microwave power is represented by zero phase difference between transmitted and reflected waves. It is observed that the number of spatial resonances increases with N_w for smaller N_p regimes whereas the spatial power follows the exponential decay law for higher N_p regimes irrespective of N_w and N_{w0}. These trends are observed for samples incident with microwaves at one face and both the faces. The heating characteristics are shown for various materials and generalized heating patterns are shown as functions of N_w, N_p and N_{w0}. The generalized heating characteristics involve either spatial temperature distributions or uniform temperature profiles based on both thermal parameters and dimensionless numbers (N_w, N_{w0}, N_p).

Keywords: Microwave; Heating; Dimensionless analysis; Material invariance

1. Introduction

Electromagnetic radiations in the frequency range 300 MHz to 300 GHz are known as microwaves (MWs). Microwaves propagate through the body and penetrate into the medium resulting in volumetric heating effect. During propagation of MWs within a material, the molecules in the material act like miniature dipoles which may orient along the direction of the electric field. The frictional energy between the dipoles while orienting towards the time-varying electric field evolves as heat energy. The heat generation due to absorption of MW power within a material is a function of the intensity of radiation, sample geometry, wavelength of the MWs and the dielectric loss of the material. Due to the volumetric heating effect, MWs offer faster thermal processing, and are largely used in chemical processing industries such as polymer processing, food processing and so on.

A large amount of earlier research on MW heating of pure substances was primarily based on experimental investigations (Ohlsson and Risman, 1978; Massoudi et al., 1979; Weil, 1975). The studies on MW heating of samples had received significant attention due to enhanced thermal effects based on specific sample dimension and material

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doi:10.1016/j.ces.2005.04.072

dielectric properties. The earlier experimental observations on enhanced heating due to MWs were observed for cylindrical and spherical substances and some of these observations include the spatial maxima in power for multilayered samples and maxima in power for spheres of specific radii over the frequency range (Ohlsson and Risman, 1978; Massoudi et al., 1979; Weil, 1975). The later research was further devoted on understanding the MW-assisted transport processes and detailed interaction between the material and MWs. The exact mechanism on combined MW and thermal transport was extensively studied by Ayappa et al. (1991, 1992) and these works include the modeling of MW-assisted heating in 1D slabs and 2D cylinders.

The modeling on MW heating is based on detailed theoretical analysis on coupled MW and heat transport for pure and multiphase samples typically used in the food industry (Ayappa et al., 1991, 1992; Zhang et al., 2001). Maxwell's equation or Lambert's exponential law for MW propagation and energy balance with a volumetric source term due to propagation on MWs form the basis of the theoretical models on MW-assisted transport processes. Ayappa et al. (1991) analyzed the heating characteristics for a multilayered food sandwich, and nonuniform or local heating was observed for 1D bread–beef food slabs. The analysis was later extended to 2D samples due to various modes on electromagnetic heating, and localized nonuniform heating was still observed for samples with specific radii (Ayappa et al., 1992).

Mathematical models of MW heating have also been developed for complex processes which take into account phenomena such as phase change (thawing and evaporation) and material property changes that occur with respect to temperature. Microwave heating and transport models were extensively studied for thawing and heating of multiphase systems in recent investigations (Basak and Ayappa, 1997, 2001, 2002; Chamchong and Datta, 1999; Basak, 2003, 2004; Lee and Marchant, 2004) and some interesting counterintuitive heating effects leading to greater rates in material processing were observed.

Extensive studies on MW processing were carried out for various materials and the heating or melting dynamics depend on the nature of material via material dielectric properties (Basak and Ayappa, 2001; Basak, 2003). The heating characteristics due to MWs are highly nontrivial and a generalized guideline on material invariant heating characteristics is important in material processing industries. Ayappa et al. (1997) and Ayappa (1999) studied MW heating characteristics for a series of materials and they observed the resonance or maxima in average power which correspond to a suitable relationship between material dimension and wavelength of MWs within the material. Their analyses are limited within the predictions of resonances which are material invariant. However, the detailed analysis on generalized power absorption and associated heating characteristics are yet to appear in the literature.

Here we carry out a detailed dimensionless analysis on the generalized characteristics of MW power and heating characteristics. The material invariant power absorption and various electric fields are expressed as functions of three dimensionless numbers: wave number (N_w), penetration number (N_p) and free space wave number (N_{w0}), where wave numbers and penetration number are the ratios of sample thickness and wavelength/penetration depth of MWs within the sample. We have shown that these three numbers uniquely govern the electric field and MW power irrespective of materials. A mathematical analysis on spatial power distributions has been carried out for 1D slabs. Microwave propagation within a sample is governed by Maxwell's equations whose solution is a linear combination of the traveling waves due to transmission and reflection. An analytical solution has been developed to study the influence of traveling waves on MW power distributions within a medium. The average power is evaluated as a function of N_w, N_p and N_{w0} and the oscillatory behavior of average power and corresponding spatial distribution are illustrated. These oscillatory distributions demonstrate the generalized features of spatial resonances of power and resonances in average power. We have also established the criteria for exponential variation of power based on Lambert's law.

Microwave heating is modeled with energy balance equation where heat generation due to MW power is a function of thermal parameters and the dimensionless numbers (N_w, N_p, N_{w0}). We have established the relationship between the spatial power distribution and dynamics of temperatures. The material invariant characteristics of temperature profile can be established based on the limits of thermal parameters and dimensionless numbers.

2. Theory

2.1. Electromagnetic field and power: a generalized dimensionless formulation

Consider a slab of thickness $2L$ exposed to MWs as shown in Fig. 1. The electromagnetic wave propagation due to a uniform electric field is governed by

$$\frac{\mathrm{d}^2 E_x}{\mathrm{d}Z^2} + k^2 E_x = 0, \tag{1}$$

where $k=(\omega/c)\sqrt{\kappa' + i\kappa''}$ is the propagation constant which depends on κ', the dielectric constant and κ'', the dielectric loss. Here, $\omega = 2\pi f$, where f is the frequency of the electromagnetic radiation and c is the velocity of light. The complex dielectric properties (κ', κ'') are often used to define two fundamental length scales of MW propagation, wavelength, λ_m, and penetration depth, D_p, as (Basak and Ayappa,